

Public capital investment and private capital productivity

Daniel Pop
PhD candidate

Supervisor: Professor László Csaba

April 2006

Abstract

The focus of the thesis is on the relationship between the long-term benefits of public infrastructure investment and economic growth under conditions of political transition and trade liberalization in Romania. We hypothesize that in contrast with a smooth transformation, sudden structural shift to open economy impacts negatively on the expected long-term returns on the public infrastructure that already existed at the start of the transition. Under these conditions, the existing infrastructure becomes oversized and/or misplaced. At the same time, however, in the longer run the systematic under- or misinvestment in infrastructure becomes a hindering factor of local firms' productivity under conditions of competitive markets.

In this excerpt, we present the formal model designed to interpret the interaction between structural change and policy reactions in terms of infrastructure provision. The proposed political economy model was built by estimating the aggregate production function of the economy, calculating public infrastructure productivity and by assuming different objective functions of policy makers when deciding upon infrastructure investments. The novelty of the model is that it considers public sector augmenting technological change and it estimates the inter- and intra-elasticity of substitution of inputs in the specified production function.

1. Introduction

Over the past fifteen years, Romania has undergone major political transition and market-oriented economic reforms. These have impacted significantly on the policy objection function of decision-makers as well as the size, structure and competitiveness of economic activities. Following a significant collapse of economic output at the beginning of the transition period and until the second part of the 1990's the recovery of the economy has been increasingly prominent, with the strengthening of new institutions. One result has been that policy-makers have increasingly faced the issue of infrastructure bottlenecks with serious hindering effects upon the expansion of the new competitive economy.

The study of political economy choices under conditions of sudden institutional changes is motivated by the fact that in contrast with a smooth transformation, the expected long-term returns on the public infrastructure that already existed at the start of the transition are subject to economic adjustment shocks as well as the possible reconfiguration of the spatial concentration of economic activity. Under these conditions, the initial infrastructure stock could become oversized compared to the level of economic activities in the early phases of transition. The costs to compensate for the depreciation of the oversized infrastructure become a major burden compared to the size of the economy. Nevertheless, in the longer run, the systematic underinvestment in infrastructure could become a hindering factor of local firms' productivity under conditions of economic take-off. Furthermore, the way in which policy-making works and the decisions on public infrastructure investments are made defines the productivity of the investments that are carried out.

The purpose of this paper is to present the political economy model built to estimate the aggregate production function of the economy, which will allow analyzing the interaction between structural change and infrastructure provision as a policy response. The proposed model seeks to calculate public infrastructure productivity by assuming different objective functions of policy makers when deciding upon infrastructure investments taking into account the level of economic activity.

This paper comprises four sections. In section 2 we discuss the literature on the productivity impact of physical infrastructure. Section 3 presents the three-level CES

production function model. The last section sums up the relevance of the model and outlines the next steps of the research.

2. Literature

Both major competing theories of regional development, i.e. neo-classical growth theory and cumulative causation theories, anticipate infrastructure investment to have important growth and development effects. In the neo-classical growth theory this effect is presented as the improvement of the working of factor markets. This theoretical conception generally relates to two different growth effects of infrastructure. On the one hand, enhanced infrastructure is hypothesized to lead to the increases of both human and physical capital, which leads to lower production costs and thus improved demand for regional output. On the other hand, infrastructure itself is a direct factor input, which de facto raises regional output. The shared view in the cumulative causation theories regarding infrastructure investment is that it contributes to development by reinforcing agglomeration economics and also constitutes into amenities.

The core difference between the two theory traits is that neo-classical growth theory supposes diminishing return on the infrastructure investment. In contrast, cumulative causation theories just predict the opposite, by seeing infrastructure investment presenting increasing returns and therefore, in a world of scarce resources and uneven levels of infrastructure development, developmental disparities will be reinforced.

The empirical ‘public infrastructure hypothesis’ has been formulated in the 1980’s. According to this thesis, the decrease in public infrastructure investment could be explained by the decline of productivity growth. The underlying concept of the hypothesis is that private and public investments are complementary to each other. The first empirical studies seeking to evaluate this relationship have focused on the productivity effects of public investment in infrastructure development. These studies were conducted by Ratner (1983) and Aschauer (1988, 1989a, 1989b). For instance, applying a production function approach, Aschauer (1989a) evaluated the output elasticity of infrastructure investment in the US, for the period 1949 to 1985, to range between 0.38 and 0.56. He found (1989a: 171-172) that public infrastructure investment contributed to the marginal productivity of private capital with more than 100 per cent and that “...an increase in non-military public capital accumulation induces as much as a dollar-for-dollar reduction in additions to the private capital stock – a complete crowding out of private by public investment expenditure”. Based on these findings, he concluded that the theoretical productivity effect of public infrastructure investment could be empirically sustained.

This was followed by a series of re-tests and further research (Garcia-Mila and MacGuire, 1992; Munnell, 1992; Munnell, 1993), which found different levels of elasticity, but all significant. In a re-test, Wylie (1996: S354), for instance, found that the productivity effects of infrastructure investment were significant and the elasticity was at the 0.407 level. Seitz (1995), using regional data of 99 German cities, found that infrastructure contributed to private output in a significant and positive way (the elasticity estimates ranged between 0.08–0.19). A more recent analysis conducted by Kolluri et al. (2000: 1067) in a time-series comparison of G7 countries found that

there was a long-run equilibrium relationship between government expenditure and national income. Thus, the author concluded that the productivity effects of infrastructure investment are long-term. All these, although in more complex ways, seem to sustain Aschauer's (1989: 172) general conclusion that "public investment policy can influence, to a dramatic extent, the course of national–public plus private–capital accumulation ...".

Lavi and Strawczynski (2002: 84) evaluate the effects of policy variables on growth in Israel. The policy variables considered include total taxes, the domestic deficit of the public sector (both expressed as a percent of GNP) and consumer price index (in terms of annual rate of change). The authors assess the effects of these variables on productivity, factors of production and output. According to their findings, the largest impact of policy variables is on factor productivity. Commonly, these production function approaches treat public infrastructure stock as one of the direct inputs of aggregate production. Nevertheless, there are authors, for instance Aaron (1990) and Tatom (1991, 1993), who debate the interpretation of Aschauer's results. For illustration, Tatom (1991, 1993) argues that it is unclear from which direction causality flows. He claims that "the hypothesis that public capital has a positive marginal private sector product cannot be supported". The author shows that the effects found by the literature are the result of disregarding the influence of energy price changes and the possibly spurious regression bias of non-stationary variables. One response to the challenges is formulated by Cadot et al. (1999) in a political-economic model, in which they conceptualize public investment in infrastructure development as endogenous in a simultaneous-equation-approach. Using panel data for French regions, the authors find political affiliation to play a significant role in

allocating public infrastructure investment capital. Also, they identify a significant relationship between the number of large firms in a given region and infrastructure investment allocation.

Erenburg (1998) took upon the criticisms formulated by Tatom (1991: 13), according to which the identified relationship between public investment in infrastructure and private capital productivity is the result of a spurious correlation. For this reason, Erenburg defined his variables in a stationary way and found that the elasticity was 0.28. Holtz and Eakin (1994) raise the issue of causality between public infrastructure investment and output growth. This re-opens a series of theoretical issues of infrastructure bottlenecks initially discussed by Hansen (1965). The work of Holtz and Eakin seeks to produce evidence suggesting that causality might run from public capital investment to output growth. Thus, the authors suggest that public infrastructure investment is a source of economic growth. Alternatively, Looney (1997) found that causality might run from growth in private output to expansion of public investment, case in which public infrastructure investment is a policy reaction to improvements in private sector output growth. Finally, Duffy-Deno and Eberts (1991: 340) found evidence that causality might run both as an unpaid factor in the production process and as consumption good for households.

All other things equal, authors using national level time-series data in a production function approach have found significantly lower elasticity levels of productivity effects of public infrastructure investment compared to studies employing cross-sectional data. This shows why, in a review article, Gramlich (1994) concludes that empirical research into the effects of public infrastructure investment on economic

growth leads to non-conclusive results. Macroeconomic analysis considering all types of infrastructure investment as having similar effects on the growth of the economy seems highly unrealistic. This explains why Barro (1989, 1990) proposes a further distinction between government provided services based on their utility-enhancing and/or productivity-enhancing feature. This contribution proved to be important for the controversy regarding the propensity of productivity enhancing effects of public infrastructure investment. Utility-enhancing public capital investment is a supplement to private capital. In other words, these types of public infrastructure investments replace private investment. This implies that the more utility-enhancing public capital investment provided, the stronger private capital crowding-out is. As a result, public capital expenditures falling into this type have a negative impact on growth. In contrast, productivity-enhancing public capital investment is a complement to private capital. Thus, any unit of public investment is conceived to attract further amounts of private investment. The more productivity-enhancing public capital investment provided, the more public capital crowds in and the growth rate improves. For instance, in the classical case of building a motorway: once built, it will reduce transport costs, and thus increase private investment to the now accessible local market.

This distinction was tested by several researchers. Thangavelu and Owyong's (2000) re-tested the infrastructure hypothesis for the case of Japan, using a sectoral approach. Their findings in what concerns the existence of a relationship between infrastructure investment and productivity effects are consistent with the results of Aschauer. Yet, they concluded that the productivity-enhancing capacity of public investment in infrastructure had significantly differentiated output effects across sectors. For the

period 1990-1995, Thangavelu and Owyong (2000: 1517) found that the output enhancing effects of public infrastructure ranged from 0.46 for paper, 0.05 for transport and 0.01 for textile industries. Yet, the large values of negative elasticity of costs to public capital show that these are utility enhancing, and thus growth inhibiting investments. This is the case of petroleum industry for all periods. Still, for 1990-1995, the value was -2.38 compared to the one of the 1989-1999 period, which was “just” -1.31 .

The above raise the issue of identifying the types of public investment in infrastructure development that have a positive productivity-enhancing effect on output growth. Then it remains to estimate the optimal level of public investment for different infrastructure goods with positive productivity effects. This problem is known as the expenditure-composition issue. The expenditure-composition problem frames the policy issue of infrastructure policy, and raises two different but interrelated issues. One is to develop a decision-making institutional framework allowing for the achievement of optimal levels of public investment in infrastructure, while the other is to allow the decision making institutional framework to be sufficiently flexible to respond effectively to variations of economic efficiency.

Taking all these into consideration, we propose to re-test the public infrastructure hypothesis in the context of political transition and economic liberalization. By dedicating the study for the analysis at the margin we expect to identify those underlying interrelationships that are likely to exist between public investment and growth.

3. Modeling infrastructure allocation under institutional change

To verify the public infrastructure hypothesis, we propose a production function with biased technological change. We assume an aggregate production function in which output (Y_t), which for simplicity is described as a function of three inputs, namely private capital (K_t), labor (L_t), and public physical infrastructure (G_t) for each year $t(1, \dots, T)$. We select the Sato (1967) type generalization of the two-factor ACMS constant elasticity of substitution (CES) production function (Arrow et al, 1961) in contrast to the ones formulated by Uzawa (1962) and McFadden (1963). The main advantage of the nested functional form proposed by Sato (1967) is that it allows for different partial elasticity of substitution among factors of production. Thus, following Sato's generalization, we define a three-level nested production function¹:

$$Y_t = F[A_t(K_t, L_t), B_t(G_t)] \quad (1)$$

To estimate the national aggregate production function, we define the variables as follows: output (Y_t) represents real GDP calculated in current prices (in USD); private capital (K_t) is the sector specific annual aggregate private capital investment, labor denotes labor services, and government services are public capital investment in road infrastructure maintenance and building, while public physical infrastructure (G_t) is described as the function of road utilization, which is calculated as the ratio between the available road infrastructure and the stock of all vehicles.

Empirical specification

The Romanian economy is presented as a production function with constant returns to scale, which is described by a three-level constant elasticity of substitution among three factor inputs². In a time-series framework, this measure provides important insights about the variance in substitutability among the three factors of production under conditions of technological change. This yields the following three-level nested CES function:

$$Y_t = \left\{ \delta_2 A_t \left[\delta_1 (K_t)^{-\alpha_{1,t}} + (1 - \delta_1) (L_t)^{-\alpha_{1,t}} \right]^{\alpha_{2,t}/\alpha_{1,t}} + (1 - \delta_2) B_t (G_t)^{-\alpha_{2,t}} \right\}^{1/\alpha_{2,t}} \quad (2)$$

where δ_1 and δ_2 are distribution parameters, A_t, B_t are efficiency parameters, and α_1 and α_2 are the intra and inter-class substitution parameters. In the following, given their importance, we will shortly discuss the last two parameters.

The efficiency parameters A_t and B_t

Most part of the literature assumes unbiased technological change among factors of production. In order to capture political transition and economic liberalization effects, we define biased efficiency parameters, which we allow to vary with different rates. Thus, we allow for temporally biased technological change, which is denoted by parameters A_t (capital/labor-augmenting efficiency) and B_t (public service-augmenting efficiency)³. For reasons of simplicity, we have assumed Hicks neutral technology change between private capital and labor inputs. By defining the index of technological efficiency in this way, we allow for factor efficiency differentials

between the private and the public decisions to invest. This, we consider, captures the specific policy distortions and adjustment of the Romanian transition economy.

When considering factor augmentation of the private economy, the literature distinguishes among three types of technological change. Factor augmenting innovation (technological change) is considered to be Hicks neutral in the situation in which the factor augmenting parameters are identical (i.e. the marginal rate of technical substitution, which means that the fraction of the marginal product of inputs is constant), resulting that inputs are not substitutes to each other. Harrod neutrality occurs when the efficiency of inputs change at different rates, but improvement is exclusively labor augmenting. In the situation when improvement is exclusively capital augmenting, we have the situation of Solow neutrality.

By putting forward a model which – besides private capital and labor inputs – also considers public inputs, we propose to include factor augmenting innovation of public inputs. This, we believe, is an effective method to move from the general normative approach of the theory of public infrastructure investment decisions to an empirical model able to capture the changes in which the public sector “innovates” its methods to carry out policy and investments in the fields of public infrastructure investments. As we see it, this approach might have the advantage to capture the rates and biases of public policy-making technological change. The applicability of this approach is especially useful in the context of major policy changes and allows for benchmarking the impact of public-augmenting technological innovation on factor efficiency and economic output. In terms of the three technological progress concepts applied to the

Sato type “three-level” CES production function, we have the following three possible scenarios:

- Hicks type neutral technological progress – the marginal rate of technical substitution between private and public inputs is constant and in order to increase output, public and private outputs need to increase at constant rates.
- Harrod type public sector input saving technological progress represents the situation in which the marginal rate of technical substitution between private and public inputs increases, meaning that private inputs are substitutes for public capital.
- Solow type private sector input saving technological progress represents the situation in which the marginal rate of technical substitution between private and public inputs decreases, implying that public capital is a substitute for private inputs.

We define the biased technological change parameters to include both economic and political indicators. The capital/labor-augmenting efficiency can be expressed as:

$$A_t = TO_t + SP_t + IC_t \quad (3)$$

The economic factors include trade openness (TO), the share of GDP delivered by private companies (SP) and investment climate (IC).

The public service-augmenting efficiency can be represented as:

$$B_t = GC_t + EC_t + RL_t \quad (4)$$

The political variables taken into consideration are the extent of “pork-barrel politics” (GC), the lack of electoral competitiveness (EC), the lack of rule of law (RL). Both efficiency parameters are defined as ordinal variables taking values from 0 to 5.

The inter- and intra-class elasticity of substitution α_1 and α_2

The main advantage of the “three-level” CES function formulated by Sato (1967, p. 205) is that it allows for different types of intra-class (α_1) and inter-class (α_2) elasticity of substitution parameters. The relationship between the two types of elasticity of substitution parameters, which are sub-unitary by definition, can be formally expressed as⁴:

- $\alpha_1, \alpha_2 = 1$ - implies that factor inputs are perfect substitutes
- $\alpha_1, \alpha_2 = -\infty$ - implies perfect complementarity among factor inputs.

Hicks (1932) introduced the elasticity of substitution concept for two-variable factor inputs. Since then several elasticity of substitution concepts have been developed in the literature seeking to generalize Hicks two factor inputs concept. The main CES concepts are the Hicks elasticity of substitution, the Allen/Uzawa elasticity of substitution and the Morishima elasticity of substitution. The main difference between the two latter elasticity concepts is that while two inputs are complements in the Allen/Uzawa CES concept, they could be either complements or supplements in the Morishima interpretation⁵. Given that the most commonly used concept is the Allen/Uzawa elasticity of substitution, we will adopt this to ensure comparability of results. Following Sato, the Allen/Uzawa specifications for inter-class elasticity of factor substitution become:

$$\sigma_2 = \sigma_{K,G} = \sigma_{L,G} = \frac{1}{1 + \alpha_2} \quad (5)$$

and the intra-class elasticity of substitution becomes:

$$\sigma_1 = \sigma_{K,L} = \frac{1}{1+\alpha_2} + \frac{1}{\gamma} \left(\frac{1}{1+\alpha_1} - \frac{1}{1+\alpha_2} \right) \quad (6)$$

where γ is the relative share of private capital and labor in total expenditure.

Now, we can verify our private-public capital complementarity hypothesis, namely that $H_0 : \sigma_2 > \sigma_1$. The complementarity hypothesis requires that:

$$\sigma_2 > \sigma_1 \Leftrightarrow \frac{1}{1+\alpha_2} + \frac{1}{\gamma} \left(\frac{1}{1+\alpha_1} - \frac{1}{1+\alpha_2} \right) < \frac{1}{1+\alpha_2} \Rightarrow \frac{1}{\gamma} \left(\frac{1}{1+\alpha_1} - \frac{1}{1+\alpha_2} \right) < 0 \quad (7)$$

which can be true iff $\alpha_2 > \alpha_1$ and $[\alpha_1 > \alpha_2]$.

Parameter testing

We assume maximization by agents and calculate the marginal products and relative share of factor inputs by taking FOC with respect to inputs, which give the following specifications of factor productivity in their stochastic form (see calculations in Appendix 3)⁶:

$$\log \frac{Y_t}{L_t} = \beta_{1,1} + \beta_{1,2} \log \frac{W_t}{P_t} - \beta_{1,3} \log A_t + \beta_{1,4} \log \gamma_t + \varepsilon_{1,t} \quad (8)$$

$$\log \frac{Y_t}{K_t} = \beta_{2,1} + \beta_{2,2} \log \frac{R_t}{P_t} - \beta_{2,3} \log A_t + \beta_{2,4} \log \gamma_t + \varepsilon_{2,t} \quad (9)$$

$$\log \frac{Y_t}{G_t} = \beta_{3,1} + \beta_{3,2} \log \frac{S_t}{P_t} - \beta_{3,3} \log B_t + \varepsilon_{3,t} \quad (10)$$

$$\log \frac{K_t}{L_t} = \beta_{4,1} + \beta_{4,2} \log \frac{W_t}{R_t} + \varepsilon_{4,t} \quad (11)$$

$$\log \frac{K_t}{G_t} = \beta_{5,1} + \beta_{5,2} \log \frac{S_t}{R_t} + \beta_{5,3} \log \frac{A_t}{B_t} + \beta_{5,4} \log \gamma_t + \varepsilon_{5,t} \quad (12)$$

$$\log \frac{L_t}{G_t} = \beta_{6,1} + \beta_{6,2} \log \frac{S_t}{W_t} + \beta_{6,3} \log \frac{A_t}{B_t} + \beta_{6,4} \log \gamma_t + \varepsilon_{6,t} \quad (13)$$

$$\log \frac{R_t}{P_t} = \beta_{7,1} + \beta_{7,2} \log \frac{Y_t}{K_t} + \beta_{7,3} \log A + \beta_{7,4} \log \gamma + \varepsilon_{7,t} \quad (14)$$

$$\log \frac{S_t}{P_t} = \beta_{8,1} + \beta_{8,2} \log Y_t + \beta_{8,3} \log B_t - \beta_{8,4} \log G_t + \varepsilon_{8,t} \quad (15)$$

$$\log \frac{W_t}{P_t} = \beta_{9,1} + \beta_{9,2} \log \frac{Y_t}{L_t} + \beta_{9,3} \log A + \beta_{9,4} \log \gamma + \varepsilon_{9,t} \quad (16)$$

In order to estimate the partial substitution elasticity among inputs, we will use time series data for the Romanian economy for the period 1990-2004.

5. Conclusions

The paper presented the formal model designed to interpret the interaction between structural change and policy reactions in terms of infrastructure provision. The novelty of the model is that it considers public sector augmenting technological change and it estimates the inter- and intra-elasticity of substitution of inputs in the specified production function. This is expected to allow us to verify the hypothesis of the thesis, namely that sudden structural shift to open economy impacts negatively on the expected long-term returns on the public infrastructure that already existed at the start of the transition and that in the longer run the systematic under- or misinvestment in infrastructure becomes a hindering factor of local firms' productivity under conditions of competitive markets.

The next stage of the research is to run the model, using a time series data set for Romania for the period 1990-2004. The data used will be annual data from the

Romanian National Bank and the National Institute of Statistics. The preliminary findings of the model suggest that: (1) in the Romanian post-socialist transition economy a significant share of the existing infrastructure stock became underused; (2) the low investment in infrastructure in the early period of transition resulted in infrastructure bottlenecks; (3) the policy responses to infrastructure bottlenecks have had a significant impact on the productivity improvements in the new economy.

Literature

- Aaron, H. J. 1990. "Discussion on D. A. Aschauer: Why is infrastructure important" in A. H. Munnell (ed.) *Is there a shortfall in public capital investment?*, Federal Reserve Bank of Boston, conference series np. 34, pp. 51-63
- Arrow, K.J., Chenery, H.B., Minhas, B.S., and Solow, R.M.. 1961, "Capital-labor substitution and economic efficiency" *Review of Economics and Statistics*, 43:225—250.
- Aschauer, D. .A. 1998. "Government spending and the falling rate of profit" *Federal Reserve Bank of Chicago, Economic Perspectives*, Vol. 12, May-June, pp. 11-17
- _____ 1989a. "Is public expenditure productive?" *Journal of Monetary Economics*, Vol. 23, pp. 177-200
- _____ 1989b. "Does Public capital crowd out private capital"? *Journal of Monetary Economics*, Vol. 24, pp. 178-235
- Cadot, O., L. H. Roller, and A Stephan. 1999. "A political economy model of infrastructure investment: An empirical Assessment". Discussion Paper FS IV 99-15, Wissenschaftszentrum Berlin
- Duffy-Deno and Eberts 1991 'Public Capital and the Factor Intensity of the Manufacturing Sector', *Urban Studies*, Volume 28, Number 1, pp. 3–14
- Erenburg, S. J. 1998. "Productivity, private and public capital, and real wage in the US", *Applied Economics Letters*, Vol. 5, pp. 491-495
- Garcia-Mila, T. and T. J. McGuire. 1992. "The contribution of publicly provided inputs to state economies", *Regional Science and Urban Economics*, Vol. 22, pp. 229-241
- Gramlich, E. M. 1994. "Infrastructure investment: A review essay" *Journal of Economic Literature*, Vol. 32, pp. 1176-1196
- Hansen, N. 1965. "The Structure and Determinants of local public investment expenditures". *Review of Economics and Statistics*, Vol. 47, pp. 150–162
- Holtz-Eakin, D. 1994," Public sector capital and the productivity puzzle", *Review of Economics and Statistics*, 76(1): 12-21.
- Kolluri, Bharat R.; Michael J. Panik and Mahmoud Substitutability Wahab. 2000. "Government expenditure and economic growth: evidence from G7 countries", *Applied Economics*, Vol. 32, pp. 1059-1068
- McFadden D. 1963. " Constant Elasticity of Substitution Production Functions," *Review Of Economic Studies*, VOL. 30, No. 2, 73-83, 1963
- Munnell, A. H. 1992. "Policy watch, infrastructure investment and economic growth", *Journal of Economic Perspectives*, Vol. 6., pp. 189-198

_____ 1993. "An assessment of trends in and economic impacts of infrastructure investment". In OECD (ed.) *Infrastructure policy for the 1990s*, pp. 21-54. Paris: OECD

Lavi, Yaakov and Michel Strawczynski. 2002. "Policy variables and growth:evidence from Israel", *Applied Economics Letters*, Vol. 9, pp. 81-86

Looney M. 1997, 'Infrastructure and private sector investment in Pakistan' *Journal of Asian Economics*, 1997, vol. 8, issue 3, pages 393-420

Sato K., 1967, "A Two-Level Constant-Elasticity-of-Substitution Production Function" *Review of Economic Studies*, Vol. 34, No. 2 (Apr., 1967) , pp. 201-218

Seitz, H. 1995: 'Public Infrastructure, Employment and Private Capital Formation,' in *The OECD Jobs Study: Investment, Productivity and Employment*, pp. 123–150. OECD, Paris.

Tatom, J. A. 1991. "Public capital and private sector performance" *Federal Reserve Bank of Saint Louis Review*, Vol. 73, pp. 3-15

_____ 1993. "Is an infrastructure crisis lowering the nations productivity?" *Federal Reserve Bank of Saint Louis Review*, Vol., 73, November/December, 3-21

Thangavelu, Shandre Mugan and David T. Owyong. 2000. "Impact of public capital on the manufacturing productive performance of Japanese industries", *Applied Economics*, Vol. 32, pp. 1515-1520

Uzawa, H. 1962 "Production Functions with Constant Elasticity of Substitution," *Review of Economic Studies* 30, 291-299.

Wylie, P. 1996. "Infrastructure and Canadian economic growth: 1946-1991", *Canadian Journal of Economics*, Vol. 29, pp. 350-355

Appendix 1 - Notation

Notation

Y_t	–	aggregate output for sector i and year t
K_t	–	private capital input for sector i and year t
L_t	–	labor input for sector i and year t
G_t	–	public physical infrastructure for year t
A_t	–	efficiency parameter for for sector i and year t
B_t	–	efficiency parameter for for sector i and year t
δ_1, δ_2	–	distribution parameters
$\alpha_{1,t}$	–	the intra-class substitution parameter for K_t and L_{it}
$\alpha_{2,t}$	–	the inter-class substitution parameter for K_t/L_t and G_t
P	–	Private capital rental price
W	–	Wage rate
S	–	Public capital rental price
$\sigma_{1,t}$	–	Represents the elasticity of substitution between K_t and L_{it}
$\sigma_{2,t}$	–	Represents the elasticity of substitution between K_t/L_t and G_t

Appendix 2 – Variables

RDINVCG	transport and communications investment central government
TSPCG	total spending central government
RDINVLC	transport and communications investment local governments
PCARS	personal cars
MCARS	merchandise motor vehicles
TOTDS	total public roads
TRANPRt	transport public roads, thousand tons
PASTR	pasangers transported, thousands
TOTINV	total investment
TEMP	total employed
TOTGDP	total capital formation
TOTEMP	total employment

Appendix 3 – Calculations

$$Y_t = \left\{ \delta_2 A_t \left[\delta_1 (K_t)^{-\alpha_{1,t}} + (1 - \delta_1) (L_t)^{-\alpha_{1,t}} \right]^{\alpha_{2,t}/\alpha_{1,t}} + (1 - \delta_2) B_t (G_t)^{-\alpha_{2,t}} \right\}^{1/\alpha_{2,t}} \quad (3)$$

where $\delta_1, \delta_2 \in (0, 1)$, $\sigma \geq 0$

Profit maximizing

$$\max_{L, K, G} PY - WL - RK - SG = P \cdot \left\{ \delta_2 A_t \left[\delta_1 (K_t)^{-\alpha_{1,t}} + (1 - \delta_1) (L_t)^{-\alpha_{1,t}} \right]^{\alpha_{2,t}/\alpha_{1,t}} + (1 - \delta_2) B_t (G_t)^{-\alpha_{2,t}} \right\}^{1/\alpha_{2,t}} - WL - RK - SG \quad (4)$$

Let for simplicity to denote the content of the $\{ \}$ bracket with Z . **Note:** for future reference we calculate

$$Y = Z^{\frac{1}{\alpha_2}} \Rightarrow Y^{-\alpha_2} = Z \Rightarrow Y^{(-\alpha_2)\left(\frac{-1}{\alpha_2}-1\right)} = Z^{-\frac{1}{\alpha_2}-1} \Rightarrow Y^{1+\alpha_2} = Z^{\frac{-1}{\alpha_2}-1} \quad (5)$$

We calculate first order conditions, which are:

$$R = P \cdot \left(-\frac{1}{\alpha_{2,t}} \right) \cdot Z^{\frac{1}{\alpha_{2,t}}-1} \cdot \delta_2 \cdot A_t \cdot \frac{\alpha_{2,t}}{\alpha_{1,t}} \cdot X^{\frac{\alpha_{2,t}-1}{\alpha_{1,t}}} \cdot \delta_1 \cdot (-\alpha_{1,t}) \cdot (K_t)^{-\alpha_{1,t}-1} = P \cdot Z^{\frac{1}{\alpha_{2,t}}-1} \cdot \delta_2 \cdot A_t \cdot X^{\frac{\alpha_{2,t}-1}{\alpha_{1,t}}} \cdot \delta_1 \cdot (K_t)^{-\alpha_{1,t}-1} \quad (6)$$

$$W = P \cdot \left(-\frac{1}{\alpha_{2,t}} \right) \cdot Z^{\frac{1}{\alpha_{2,t}}-1} \cdot \delta_2 \cdot A_t \cdot \frac{\alpha_{2,t}}{\alpha_{1,t}} \cdot X^{\frac{\alpha_{2,t}-1}{\alpha_{1,t}}} \cdot (1 - \delta_1) (-\alpha_{1,t}) \cdot (L_t)^{-\alpha_{1,t}-1} = P \cdot Z^{\frac{1}{\alpha_{2,t}}-1} \cdot \delta_2 \cdot A_t \cdot X^{\frac{\alpha_{2,t}-1}{\alpha_{1,t}}} \cdot (1 - \delta_1) \cdot (L_t)^{-\alpha_{1,t}-1} \quad (7)$$

$$S = P \cdot \left(-\frac{1}{\alpha_{2,t}} \right) \cdot Z^{\frac{1}{\alpha_{2,t}}-1} \cdot (1 - \delta_2) \cdot (-\alpha_{2,t}) \cdot B_t \cdot (G_t)^{-\alpha_{2,t}-1} = P \cdot Z^{\frac{1}{\alpha_{2,t}}-1} \cdot (1 - \delta_2) \cdot B_t \cdot (G_t)^{-\alpha_{2,t}-1} \quad (8)$$

Now, we can replace equation (5) in these results, which gives:

$$R_t = P_t \cdot Y_t^{1+\alpha_2} \cdot \delta_2 \cdot A_t \cdot X_t^{\frac{\alpha_2}{\alpha_1}-1} \cdot \delta_1 \cdot K_t^{-\alpha_1-1} \quad (9)$$

$$W_t = P_t \cdot Y_t^{1+\alpha_2} \cdot \delta_2 \cdot A_t \cdot X_t^{\frac{\alpha_2}{\alpha_1}-1} \cdot (1 - \delta_1) \cdot L_t^{-\alpha_1-1} \quad (10)$$

$$S_t = P_t \cdot Y_t^{1+\alpha_2} \cdot (1 - \delta_2) \cdot B_t \cdot G_t^{-\alpha_2-1} \quad (11)$$

The marginal product of private capital and labor is calculated by dividing (9) and (10). Making all the simplifications and taking logarithms give:

$$\log \frac{R_t}{W_t} = \log \frac{\delta_1}{1 - \delta_1} + (\alpha_1 + 1) \log \frac{L_t}{K_t} \quad (12)$$

Now, if we replace X in equation (3), do the division according to (12) and multiply it with $(1 - \delta_1) \cdot L^{-\alpha_1}$, we get:

$$X_t = \left[\frac{\delta_1}{1 - \delta_1} \cdot \left(\frac{K_t}{L_t} \right)^{-\alpha_1} + 1 \right] \cdot (1 - \delta_1) \cdot L_t^{-\alpha_1} \Rightarrow X_t^{\frac{\alpha_2 - 1}{\alpha_1}} = X_t^{\frac{\alpha_2 - \alpha_1}{\alpha_1}} = \left[\frac{\delta_1}{1 - \delta_1} \cdot \left(\frac{K_t}{L_t} \right)^{-\alpha_1} + 1 \right]^{\frac{\alpha_2 - 1}{\alpha_1}} \cdot (1 - \delta_1)^{\frac{\alpha_2 - 1}{\alpha_1}} \cdot L_t^{\alpha_1 - \alpha_2} \quad (13)$$

Similarly, we replace X from (3), do the division according to (12) and multiply it with $\delta_1 \cdot K^{-\alpha_1}$, we get:

$$X_t^{\frac{\alpha_2 - 1}{\alpha_1}} = \left[1 + \frac{1 - \delta_1}{\delta_1} \cdot \left(\frac{L_t}{K_t} \right)^{-\alpha_1} \right]^{\frac{\alpha_2 - 1}{\alpha_1}} \cdot (\delta_1)^{\frac{\alpha_2 - 1}{\alpha_1}} \cdot K_t^{\alpha_1 - \alpha_2} \quad (14)$$

We can replace (14) in (6) and rearrange, which gives:

$$\frac{R_t}{P_t} = \left(\frac{Y_t}{K_t} \right)^{1 - \alpha_2} \cdot \delta_2 \cdot (\delta_1)^{\frac{\alpha_2}{\alpha_1}} \cdot A_t \cdot \left[1 + \frac{1 - \delta_1}{\delta_1} \cdot \left(\frac{L_t}{K_t} \right)^{-\alpha_1} \right]^{\frac{\alpha_2 - 1}{\alpha_1}} \quad (15)$$

We can replace (13) in (7) and rearrange, which gives:

$$\frac{W_t}{P_t} = \left(\frac{Y_t}{L_t} \right)^{1 + \alpha_2} \cdot \delta_2 \cdot (1 - \delta_1)^{\frac{\alpha_2}{\alpha_1}} \cdot A_t \cdot \left[\frac{\delta_1}{1 - \delta_1} \cdot \left(\frac{K_t}{L_t} \right)^{-\alpha_1} + 1 \right]^{\frac{\alpha_2 - 1}{\alpha_1}} \quad (16)$$

We use (15) and (16) to calculate (8):

$$\left(\frac{K_t}{L_t} \right)^{-\alpha_1} = \left(\frac{R_t}{W_t} \cdot \frac{1 - \delta_1}{\delta_1} \right)^{\frac{\alpha_1}{1 + \alpha_1}} \quad (17)$$

$$\left(\frac{L_t}{K_t} \right)^{-\alpha_1} = \left(\frac{R_t}{W_t} \cdot \frac{1 - \delta_1}{\delta_1} \right)^{-\frac{\alpha_1}{1 + \alpha_1}} \quad (18)$$

We can replace (17) in (16), make the calculation, and take logarithms, which gives:

$$\log \frac{Y_t}{L_t} = \frac{1}{1 + \alpha_2} \log \frac{W_t}{P_t} - \frac{1}{1 + \alpha_2} \log A_t + \frac{1}{1 + \alpha_2} \log \frac{1}{\delta_2} + \frac{\alpha_2}{\alpha_1(1 + \alpha_2)} \log \frac{1}{1 - \delta_1} + \frac{\alpha_1 - \alpha_2}{\alpha_1(1 + \alpha_2)} \log \left[1 + \left(\frac{R_t}{W_t} \right)^{\frac{\alpha_1}{1 + \alpha_1}} \cdot \left(\frac{\delta_1}{1 - \delta_1} \right)^{\frac{1}{1 + \alpha_1}} \right] \quad (19)$$

We replace (18) in (15) and take logarithms, which gives:

$$\log \frac{Y_t}{K_t} = \frac{1}{1 + \alpha_2} \log \frac{R_t}{P_t} - \frac{1}{1 + \alpha_2} \log A_t + \frac{1}{1 + \alpha_2} \log \frac{1}{\delta_2} + \frac{\alpha_2}{\alpha_1(1 + \alpha_2)} \log \frac{1}{\delta_1} + \frac{\alpha_1 - \alpha_2}{\alpha_1(1 + \alpha_2)} \log \left[1 + \left(\frac{R_t}{W_t} \right)^{-\frac{\alpha_1}{1 + \alpha_1}} \cdot \left(\frac{1 - \delta_1}{\delta_1} \right)^{\frac{1}{1 + \alpha_1}} \right] \quad (20)$$

We can rewrite (11) and take logarithms, which gives:

$$\log \frac{Y_t}{G_t} = \frac{1}{1+\alpha_2} \log \frac{S_t}{P_t} + \frac{1}{1+\alpha_2} \log \frac{1}{B_t} + \frac{1}{1+\alpha_2} \log \frac{1}{1-\delta_2} \quad (21)$$

We calculate the marginal product of private capital and public capital by dividing (9) with (11), take (14) for $X_t^{\frac{\alpha_2-1}{\alpha_1}}$ according to (18), and take logarithms, which results:

$$\log \frac{K_t}{G_t} = \frac{1}{\alpha_2-1} \log \frac{S_t}{R_t} + \frac{1}{\alpha_2-1} \log \frac{\delta_2}{1-\delta_2} + \frac{1}{\alpha_2-1} \log \frac{A_t}{B_t} + \frac{\alpha_2}{\alpha_1(\alpha_1+1)} \log \delta_1 + \frac{\alpha_2-\alpha_1}{\alpha_1(\alpha_2-1)} \log \left[1 + \left(\frac{R_t}{W_t} \right)^{\frac{\alpha_1}{1+\alpha_1}} \cdot \left(\frac{1-\delta_1}{\delta_1} \right)^{\frac{1}{1+\alpha_1}} \right] \quad (22)$$

We divide (10) with (11), use (13) for $X_t^{\frac{\alpha_2-1}{\alpha_1}}$ according to (17), and take logarithms, which gives:

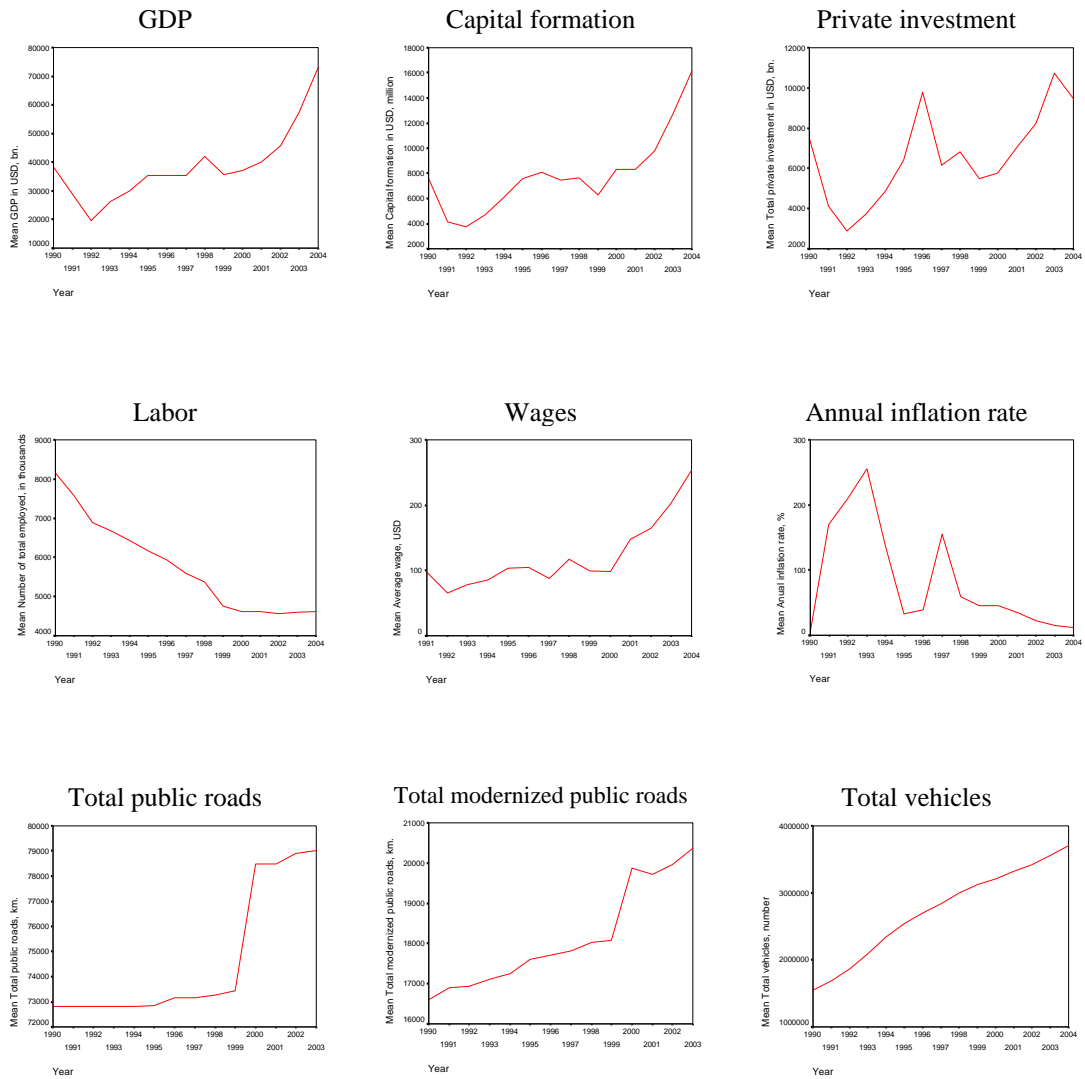
$$\log \frac{L_t}{G_t} = \frac{1}{\alpha_2+1} \log \frac{S_t}{W_t} + \frac{1}{\alpha_2+1} \log \frac{\delta_2}{1-\delta_2} + \frac{1}{\alpha_2+1} \log \frac{A_t}{B_t} + \frac{\alpha_2}{\alpha_1(\alpha_2+1)} \log(1-\delta_1) + \frac{\alpha_1-\alpha_2}{\alpha_1(\alpha_2+1)} \log \left[1 + \left(\frac{R_t}{W_t} \right)^{\frac{\alpha_1}{1+\alpha_1}} \cdot \left(\frac{\delta_1}{1-\delta_1} \right)^{\frac{1}{1+\alpha_1}} \right] \quad (23)$$

Case Summaries^a

	GDP in USD, bn.	Number of total employed, in thousands	Average wage, USD	Total private investment in USD, bn.	Capital formation in USD, million	Total vehicles, number	Total public roads, km.	
1	38247,88	8156	,	7507,80	7570,22	1550984	72816	
2	28850,63	7574	97,66	4110,49	4149,76	1691132	72816	
3	19578,50	6888	65,40	2885,53	3756,78	1868516	72816	
4	26361,03	6672	78,57	3712,65	4715,08	2091372	72816	
5	30072,81	6438	85,77	4836,35	6099,79	2342434	72828	
6	35477,75	6160	103,96	6391,46	7586,29	2540541	72859	
7	35333,68	5939	104,19	9794,69	8109,55	2702994	73160	
8	35285,69	5597	88,18	6157,24	7469,38	2837268	73161	
9	42115,50	5369	117,43	6818,19	7652,47	3000314	73260	
10	35592,04	4761	99,32	5475,02	6302,15	3119851	73435	
11	37052,63	4623	98,61	5761,71	8298,23	3204746	78479	
12	40180,74	4619	147,37	7026,47	8298,23	3319159	78492	
13	45824,53	4568	164,94	8220,57	9766,50	3420689	78892	
14	57329,82	4591	203,05	10738,29	12726,93	3551627	79001	
15	73166,83	4620	253,14	9478,82	16133,92	3707000	,	
Total	N	15	15	14	15	15	14	
	Median	35592,04	5597,00	101,6400	6391,4600	7586,2900	2837268,00	73160,50
	Grouped Median	35592,04	5597,00	101,6400	6391,4600	7586,2900	2837268,00	73160,50
	Std. Error of Mean	3334,4667	305,85	14,0872	589,3842	823,9349	178910,20	719,32
	Sum	580470,06	86575	1707,59	98915,28	118635,28	40948627	1044831
	Minimum	19578,50	4568	65,40	2885,53	3756,78	1550984	72816
	Maximum	73166,83	8156	253,14	10738,29	16133,92	3707000	79001
	Range	53588,33	3588	187,74	7852,76	12377,14	2156016	6185
	First	38247,88	8156	97,66	7507,80	7570,22	1550984	72816
	Last	73166,83	4620	253,14	9478,82	16133,92	3707000	79001
	Std. Deviation	12914,33	1184,55	52,7095	2282,6751	3191,0860	692916,22	2691,44
	Variance	1,7E+08	1403167,810	2778,295	5210605,725	10183030,08	4,8013E+11	7243841,26
	Kurtosis	2,975	-,661	1,882	-,594	2,320	-,1085	-,1008
	Std. Error of Kurtosis	1,121	1,121	1,154	1,121	1,121	1,121	1,154
	Skewness	1,424	,636	1,524	,245	1,287	-,375	1,056
	Std. Error of Skewness	,580	,580	,597	,580	,580	,580	,597
	Harmonic Mean	35331,69	5562,37	106,9331	5801,4158	6888,2954	2541015,88	74543,50
	% of Total Sum	*****	1,322+292%	2,61+290%	1,510+292%	1,811+292%	6,251+294%	*****
	% of Total N	*****	2,304+288%	2,15+288%	2,304+288%	2,304+288%	2,304+288%	*****

a. Limited to first 100 cases.

The evolution of the main variables used in the model



¹ In the literature there is disagreement over the notation of nests. For simplicity, we subscribe to the one adopted by Sato (1967). This is motivated by the fact that our interest is to evaluate both intra- and inter-elasticity of substitution.

² The functional form specification of the production function in the case of applied general-equilibrium models in the literature is divided among: Cobb-Douglas, Constant Elasticity of Substitution (CES), Leontief production functions, and Translog function. The most commonly used are the Cobb-Douglas functional production functions, mostly because of the simplicity of modeling. The main weakness of Cobb-Douglas is its strong substitutability of inputs. To correct this, ACMS developed the CES production functions, which allows for the elasticity of substitution to be different from one.

³ We have defined two technology parameters in order to underline and capture the differences in the market structures of the private and public utility maximization.

⁴ The most studied Cobb-Douglas specification corresponds to the particular case in which $\alpha_1, \alpha_2 = 0$.

⁵ For the formal theoretical discussion of the different CES concepts, please see: Blackorby, Charles and Robert R. Russell (1989) "Will the Real Elasticity of Substitution Please Stand Up? (A Comparison of the Allen/Uzawa and Morishima Elasticities)" *The American Economic Review*, vol. 79, no. 4 (sept., 1989), pp. 882-888, and Mundra, Kusum and Robert R. Russell (2004) "Dual Elasticities of Substitution", <<http://www.economics.ucr.edu/people/russell/01-26.pdf>> (accessed October 18, 2005).

⁶ We replaced with γ_t the relative share of the factors in the nested function (K, L) , which is

$$\left[1 + \left(\frac{R_t}{W_t} \right)^{\frac{\alpha_1}{1+\alpha_1}} \left(\frac{\delta_t}{1-\delta_t} \right)^{\frac{1}{1+\alpha_1}} \right]$$