1) BASIC ALGEBRA 1

Course Coordinator: Peter Hermann

Prerequisites: Introductory linear algebra course and a two-semester undergraduate course in abstract algebra.

Books: N. Jacobson, Basic algebra I-II, W. H. Freeman and Co., San Francisco 1974/1980.

Commitment: 3 hours/week, 3 credits
Contents:
Polynomials (Gauss Lemma, cyclotomic polynomials, polynomials in several variables, homogeneous polynomials, symmetric polynomials, formal power series, Newton's Formulas, twisted polynomials, polynomials in non-commuting variables). Groups (free groups, generators and defining relations, commutator subgroup, solvable groups, simple groups, simplicity of the alternating groups, classical linear groups). Rings and modules (simplicity of matrix rings, quaternions, Frobenius Theorem, submodules, homomorphisms, direct sums of modules, free modules). Partially ordered sets and lattices (Hasse-diagram, chain conditions, Zorn Lemma, lattices as posets and as algebraic structures, modular and distributive lattices, modularity of the lattice of normal subgroups, Boolean algebras, Stone Representation Theorem). Universal algebra (subalgebras, homomorphisms, direct products, varieties, Birkhoff Theorem, subdirect products, subdirectly irreducible algebras, subdirect representation). Categorical approach (products, coproducts, pullback, pushout, functor categories, natural transformations, Yoneda lemma, adjoint functors).

2) BASIC ALGEBRA 2

Course Coordinator: Laszlo Marki
Prerequisites: Basic Algebra 1
Books:
N. Jacobson, Basic algebra I-II, W. H. Freeman and Co., San Francisco 1974/1980.
Commitment: 3 hours/week, 3 credits
Contents:

- Groups (composition series, Jordan-Hölder Theorem, conjugation, centralizer, normalizer, class equation, p-groups, nilpotent groups, Frattini subgroup, Frattini argument, direct product, Krull-Schmidt Theorem, semidirect product, groups of small order).
- Commutative rings (unique factorization, principal ideal domains, Euclidean domains, finitely generated modules over principal ideal domains, Fundamental Theorem of finite abelian groups, Jordan normal form of matrices, Noetherian rings, Hilbert Basis Theorem, operations with ideals).
- Fields (algebraic and transcendental extensions, transcendence degree, splitting field, algebraic closure, the Fundamental Theorem of Algebra, normal extensions, finite fields, separable extensions, Galois group, Fundamental Theorem of Galois Theory, cyclotomic fields, radical expressions, insolvability of the quintic equation, traces and norms: Hilbert and Artin-Schreier theorems, ordered and formally real fields).

3) BASIC ALGEBRA 3

Course Coordinator: Pham Ngoc Anh
Prerequisites: Basic Algebra 1, 2
Books:
1. N. Jacobson, Basic algebra I-II, W. H. Freeman and Co., San Francisco 1974/1980.2. J. Serre, Local algebra, Springer-Verlag, Berlin 2000
Commitment: 3 hours/week, 3 credits

Contents:
- Commutative rings (prime ideals, radical of an ideal, Krull's theorems, McCoy's theorem, primary decomposition for modules, Lasker-Noether Theorem, Krull Intersection Theorem, Krull dimension, integral extensions, Dedekind domains, Noether Normalization Lemma, Hilbert Nullstellensatz, valuations, classical elimination theory and Bezout theorem).
- Non-commutative rings (Jacobson radical, Jacobson density theorem, Artinian rings, semisimple rings and algebras, Wedderburn-Artin Theorem, projective and injective modules, tensor product of modules, Lie and Jordan algebras).
- Group representations (modules over a group algebra, irreducible representations, Maschke Theorem, characters, orthogonality relations, applications to centrally simple algebras and Brauer groups).

4) REAL ANALYSIS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Undergraduate Calculus, Elementary Linear Algebra
Books:
1. J.E. Marsden and M.J. Hoffman, Elementary Classical Analysis, Second Edition, W.H. Freeman & Co., New York, 1993.
2. M.H. Protter and C.B. Morrey, A First Course in Real Analysis, Second Ed., Springer-Verlag, 1977.
3. Walter Rudin, Principles of Mathematical Analysis, Third Edition, McGraw-Hill, Inc., 1964.

Commitment: 3 hours/week, 3 credits

Contents:
- The real number system
- Basic topology
- Numerical sequences and series
- Limits, continuity, and differentiation
- The Riemann and Riemann-Stiltjes integrals
- Sequences and series of functions
- Functions of several real variables
- Vector field theory. The divergence theorem
- The Lebesgue integral . Lp-spaces
- Fourier analysis
- Distributions

5) COMPLEX FUNCTION THEORY

Course Coordinator: Gabor Halasz
Prerequisites: Undergraduate Calculus
Books:
1. Ahlfors, L., Complex Analysis, McGraw-Hill, 1979.
2. Rudin, W., Real and Complex Analysis, McGraw-Hill, 1987.

Commitment: 3 hours/week, 3 credits

Contents:
Complex numbers, analytic functions, Cauchy's theorem and consequences, isolated singularities, analytic continuation, open mapping theorem, infinite series and products, harmonic and subharmonic functions, maximum principle, fractional linear transformations, geometric and local properties of analytic functions, Weierstrass Theorem, normal families, residues, conformal mapping, Riemann mapping theorem, branch points, second order linear O.D.E.'s., the Riemann zeta function.

6) FUNCTIONAL ANALYSIS AND DIFFERENTIAL EQUATIONS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Linear Algebra, Calculus, Real and Complex Analysis
Books:
1. H. Brezis, Analyse fonctionnelle. Theorie et applications, Masson, Paris, 1983.
2. A. Pazy, Semigroups of Linear Operators and Applications to PDEs, Springer-Verlag, 1983.
3. E. Zeidler, Applied Functional Analysis, Appl. Math. Sci. 108,109, Springer-Verlag, 1995.

Commitment: 3 hours/week, 3 credits

Contents:
- Normed linear spaces. Banach spaces.
- Fixed-point theorems and applications to differential and integral equations
- The Hahn-Banach theorem
- The main principles of linear functional analysis
- Weak topologies. Reflexive and separable spaces
- Hilbert spaces
- The Riesz representation theorem. The Lax-Milgram theorem
- Orthonormal systems in Hilbert spaces
- Eigenvalue problems for linear compact operators. The Hilbert-Schmidt theory. The Fredholm alternative. Applications
- Semigroups of linear operators. The Hille-Yosida and Lumer-Phillips theorems
- Linear evolution equations in Banach spaces and applications to PDE

7) ENUMERATION

Course Coordinator: Zoltán Füredi
Prerequisites: Binomial coefficients, binomial theorem, permutations, variations, Fibonacci numbers
Books:

1. D.E. Knuth, The Art of Computer programming, Third Edition (Reading, Massachusetts: Addison-Wesley, 1997.
2. R.L. Graham, D.E. Knuth, O. Patashnik: Concrete Mathematics: a Foundation for Computer Science, Addison-Wesley, Reading, U.S.A., 1989.
3. H.S. Wilf: Generatingfunctionology, Academic Press, 1990.
4. G.E. Andrews, The theory of partitions, Addison-Wesley, 1976.

Commitment: 3 hours/week, 3 credits

Contents:

- Binomial Theorem, Polynomial Theorem. Stirling Formula. Partitions of an integer. Fibonacci numbers. Counting examples from geometry and Information Theory. Generating Functions Linear congruences. Fibonnacci numbers. Recurrences. Inversion formulas. Partitions of sets and numbers Catalan numbers. Young Tableaux Cayley Theorem.
- Rényi's examples to count trees. Asymptotic series. Watson Lemma. Saddle point method. Stirling formula.
- Inclusion-Exclusion formulas: Sieve Method. Möbius function, Möbius inversion formula Applications in number theory. Pólya Method.
- Using computers. Wilf-Zeilberger theory.

8) EXTREMAL COMBINATORICS

Course Coordinator: Miklós Simonovits
Prerequisites: Some introductory course to discrete mathematics, e.g. Introduction to graph theory and combinatorics.
Books:

1. B. Bollobás: Extremal Graph Theory, Academic Press, London, 1978.
2. R. L. Graham, B. L. Rothschild, and J. H. Spencer, Ramsey theory, Wiley, New York, 1980.

Commitment: 3 hours/week, 3 credits

Contents:

- Ramsey Theory.
  The Erdős-Szekeres estimate. Hypergraph Ramsey Theorems. Van der Waerden theorem. Hales-Jewett theorem. Amalgamation method (Nesetril-Rödl)
- Extremal Graph Theory.
  Turán's theorem. Erdős-Stone-Simonovits theorem on the limit density. Non-degenerate extremal graph problems. Asymptotic structure of extremal graphs. Kővári-T. Sós-Turán theorem. Constructions. Füredi's theorem on fourgous. Degenerate extremal graph problems. Erdős-Gallai Theorem.
- Supersaturated graphs
- Szemerédi Regularity Lemma
- Extremal graph problems for uniform hypergraphs Ruzsa-Szemerédi theorem. The Szemerédi theorem on arithmetic progressions.

9) RANDOM METHODS IN COMBINATORICS

Course Coordinator: Vera T. Sós
Prerequisites: Random fields, independence, pairwise independence, conditional probability, binomial coefficients, Stirling formula
Books:
1. N. Alon, J.H. Sepncer: The Probabilitstic Method, John Wiley & Sons, 1992.
2. B. Bollobás: Random Graphs, Academic Press, 1985.
3. P. Erdős: The Art of Counting, Cambridge, MIT Press, 1973.
4. P. Erdős: Joel Spencer: Probabilitstic Methods in Combinatorics, Academic Press, 1974.
5. J. Matousek, Geometric Discrepancy, Algorithms and Combinatorics 18, Springer, 1999.

Commitment: 3 hours/week, 3 credits
Contents:
Global Description: Tools from probability theory, application of random and modified random methods to prove existence results, typical objects, evolution of random structures.
- Lovász Local Lemma, applications. Chernov inequality. FKG inequality, Janson inequality, Boppana-Spencer proof. Derandomization of randomized constructions and Regularity Lemma
- Random Ramsey construction. Disproof of Hajós' conjecture. Random construction of graphs with high chromatic number and large girth. Pseudorandom constructions. Heilbronn's problem
- Applications in discrepancy theory
- Random Structures
  Random Regular graphs. The evolution of random graphs. Hamilton lines/cycles in random graphs. Chromatic number of random graphs
- Pseudorandom and Quasirandom structures.

10) CONVEX GEOMETRY

Course Coordinator: Balázs Csikós
Prerequisites: abstract and linear algebra, general topology, analysis
Books: 1. M. Berger, Geometry I-II, Springer-Verlag, New York, 1987.
2. K.W. Gruenberg and A.J. Weir, Linear Geometry, Springer, 1977.

Commitment: 3 hours/week, 3 credits
Contents:
- Affine spaces.
- Euclidean space, structure of the isometry group, canonical form of isometries, Cartan's theorem.
- Spherical trigonometry.
- Fundamental theorems on convex sets (Caratheodory, Radon, Helly, Krein-Milman, Straszewicz etc.).
- Convex polytopes, Euler's formula, classification of regular polytopes, Cauchy's rigidity theorem, flexible polytopes.
- Hausdorff metric, Blaschke's selection theorem, Cauchy's formula, the Steiner-Minkowski formula, symmetrizations, isoperimetric and isodiametral inequalities.

11) NON-EUCLIDEAN GEORMETRIES

Course Coordinator: Balázs Csikós
Prerequisites: abstract and linear algebra, general topology, analysis
Books:
1. M. Berger, Geometry I-II, Springer-Verlag, New York, 1987.
2. K.W. Gruenberg and A.J. Weir, Linear Geometry, Springer, 1977.

Commitment: 3 hours/week, 3 credits

Contents:
- Projective spaces over division rings, Desargues' and Pappus' theorem, axiomatic foundations, the duality principle.
- Collineations, correlations, cross-ratio preserving transformations.
- Quadrics, classification of quadrics, Pascal's and Brianchon's theorems, polarity induced by a quadric, pencils of quadrics, Poncelet's theorem.
- Hyperbolic geometry: models of the hyperbolic space and the transition between them, isometries, hyperbolic trigonometry, constructions.

12) DIFFERENTIAL GEOMETRY

Course Coordinator: Balázs Csikós
Prerequisites: abstract and linear algebra, general topology, analysis, ordinary differential equations
Books:
1. M.P. do Carmo: Differential Geometry of Curves and Surfaces Prentice-Hall, Englewood Cliffs, NJ, 1976.
2. W. Klingenberg: A course in differential geometry, Springer, 1978.
3. W.M. Boothby: An introduction to differentiable manifolds and Riemannian geometry, Second Edition, Academic Press, 1986.

Commitment: 3 hours/week, 3 credits

Contents:
- Curves in $\boldsymbol{R}^2$.
- Hypersurfaces in $\boldsymbol{R}^2$. Theorema Egregium. Special surfaces.
- Differentiable manifolds, tangent budle, tensor bundles; Lie algebra of vector fields, distributions and Frobenius' theorem; Covariant derivation, the Levi-Civita connection of a Riemannian manifold, parallel transport, holonomy groups; Curvature tensor, symmetries of the curvature tensor, decomposition of the curvature tensor; Geodesic curves, the exponential map, Gauss Lemma, Jacobi fields, the Gauss-Bonnet theorem; Differential forms, de Rham cohomology, integration on manifolds, Stokes' theorem.

13) HOMOLOGICAL ALGEBRA

Course Coordinator: Pham Ngoc Anh
Prerequisites: Basic algebra 1-3
Books:
C. Weibel, An introduction to homological algebra, Cambridge Univ. Press, 1996
Commitment: 3 hours/week, 3 credits
Contents:

- Language of categories and functors. The Hom-functors and the tensor product functor, adjunction formula. Exactness. Limits, diagrams.
- Projective, injective and flat modules. Resolutions and co-resolutions. Representation theorems of Watts.
- Homological properties of specific ring classes (semisimple, Noetherian, Artinian, polynomial rings etc.). Serre's Conjecture: the Quillen-Suslin theorem. Examples from topology (singular, simplicial homology etc.). Group extensions.
- Category of chain complexes; homology groups. Quasi-isomorphisms of chain complexes. Chain homotopies. Mapping cones and cylinders. Long exact sequences of homology groups.
- Derived functors: the construction of Ext and Tor; basic properties, the Yoneda product. Koszul-duality. Ext and Tor for specific classes of rings. Universal Coefficient Theorems.
- Homological dimensions: projective, injective, global, finitistic dimension, etc. (Hilbert's Syzygy Theorem; the Finitistic Dimension Conjecture).
- Applications: singular homology in topology, dimension theory of commutative rings, group cohomology, representation theory of associative Artin algebras (Auslander—Reiten theory), sheaf cohomology.

14) SMOOTH MANIFOLDS AND DIFFERENTIAL TOPOLOGY

Course Coordinator: András Némethi
Prerequisites: Basic topology, algebra and analysis
Books:

1. J. W. Milnor: Topology from the Differentiable Viewpoint, Princeton University Press, 1997.
2. I. Madsen and J. Tornehave: From Calculus to Cohomology, De Rham cohomology and characteristic classes, Cambridge University Press, 1997.

Commitment: 3 hours/week, 3 credits
Contents:

- Differentiable manifolds, tangent space, cotangent space, tensors;
- Differential forms in orientation of manifolds, integration on manifolds;
- Stokes Theorem, Poincaré Lemma, De Rham cohomology;
- The Mayer-Vietoris sequence, examples;
- Sard's theorem, Morse functions, Morse theory;
- Degree, intersection index, linking number, Poincaré duality;
- The index of a vector field, Euler number, Poincaré-Hoph Theorem;
- Fiber bundles and vector bundles;
- Characteristic classes, Thom isomorphism;
- Spectral sequences and applications.

15) ALGEBRAIC TOPOLOGY

Course Coordinator: András Stipsicz
Prerequisites: Basic topology
Books:
A. Hatcher: Algebraic Topology, Cambridge University Press, 2002.

Commitment: 3 hours/week, 3 credits

Contents:
- Fundamental groups, some examples.
- Covering spaces, relations to fundamental groups. Universal covers. Deck transformations.
- Van Kampen's theorem. Fundamental groups of 1- and 2-complexes.
- Homotopy groups, their relation to coverings.
- Simplicial complexes, simplicial homology. Singular homology.
- Mayer-Vietoris sequence. Axioms for homology, categories and functors.
- Cohomology. The Universal Coefficient Theorem.
- Cup product. Manifolds and orientations.
- Poincaré duality. Künneth formula.
- Applications

16) FUNCTION SPACES AND DISTRIBUTIONS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Undergraduate Calculus, Linear Algebra, Linear Functional Analysis

Books:
1. R.A. Adams, Sobolev Spaces, Academic Press, 1975.
2. H. Brezis, Analyse fonctionelle,. Theorie et applications, Masson, Paris, 1983.
3. I.M. Gel'fand and G.E. Shilov, Generalized Functions, Vols 1 and 2, Academic Press, 1968.
4. L. Schwartz, Theorie de distributions, Herman, Paris, 1967.

Commitment: 3 hours/week, 3 credits

Contents:
- Spaces of continuous functions
- Functions of bounded variation
- The spaces $L^p(\Omega)$, $L^p(a,b; X)$
- Test functions and distributions
- Density theorems
- Sobolev spaces
- The spaces $W^{k,p}(a,b; X)$
- Applications

17) NONLINEAR FUNCTIONAL ANALYSIS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Real and Complex Analysis, Linear Functional Analysis
Books:
1. G. Morosanu, Nonlinear Evolution Equations and Applications, Reidel, 1988.
2. E. Zeidler, Applied Functional Analysis. Applications to Mathematical Physics, Applied Math. Sci. 108, Springer-Verlag, 1995

Commitment: 3 hours/week, 3 credits

Contents:
- Fixed point theorems. Applications
- Variational principles and weak convergence. The n-th variation. Necessary and sufficient conditions for local extrema. Weak convergence. The generalized Weierstrass existence theorem. Applications to calculus of variations. Applications to nonlinear eigenvalue problems. Applications to convex minimum problems and variational inequalities. Applications to obstacle problems in Elasticity. Saddle points. Applications to duality theory. The von Neumann Minimax theorem on the existence of sadle points. Applications to game theory.
- Nonlinear monotone operators. Applications.

18) INTRODUCTION TO MATHEMATICAL LOGIC

Course Coordinator: Sain Ildikó
Prerequisites: Elements of set theory.

Books:
1. H. B. Enderton: A mathematical introduction to logic. Academic Press, New York and London, 1972.
2. H. D. Ebbinghaus, J. Flum and W. Thomas: Mathematical logic. Springer-Verlag, Berlin, 1984.

Commitment: 3 hours/week, 3 credits

Contents:
- Sentential (propositional) logic. Syntax and semantics. Completeness, compactness, decidability. Connections with Boolean algebras.
- First order logic: syntax and semantics
- A deductive calculus. Soundness and completeness theorems.
- Ultraproducts. Los lemma.
- Compactness theorem, Löwenheim-Skolem theorems, preservation theorems; complete theories, decidable theories; applications.
- Elementary classes. Elementarily equivalent structures.
- Gödel's incompleteness theorems.
- Definability.
- Many-sorted logics.
- Higher order logics. Incompleteness of second order logic. Absoluteness and completeness of logics. Absolute versions of higher order logics.

19) MODERN SET THEORY

Course Coordinator: István Juhász
Prerequisites: Introduction to Mathematical Logic
Books:

1. András Hajnal, Peter Hamburger: Set Theory, Cambridge University Press, 1999.
2. Thomas Jech: Set Theory, Spinger-Verlag, 1997.
3. Kenneth Kunen: Set theory. An introduction to Independence Proofs, Elsevier, 1999.

Commitment: 3 hours/week, 3 credits

Contents:

- The cumulative hierarchy and the ZFC axiom system
- Axiomatic exposition of set-theory
- Absoluteness and reflection
- Models of set-theory, relative consistency
- Constructible sets, consistency of AC and GCH
- Combinatorial set-theory and combinatorial principles
- Large cardinals
- Basic forcing

20) ALGEBRAIC LOGIC AND MODEL THEORY

Course Coordinator: Gábor Sági
Prerequisites: Universal Algebra course
Books:

1. Henkin, L. Monk, J. D. and Tarski, A.: Cylindric Algebras. Parts I-II., North-Holland, 1985.
2. Chang, C. C. and Keisler, H. J.: Model Theory. North-Holland, Amsterdam, 1994.
3. Barwise, J. and Fefermann, S.: Model Theoretic Logics. Elsevier, 1986.

Commitment: 3 hours/week, 3 credits

Contents:

- Algebras of logic: Boolean algebras, cylindric algebras, quasi-polyadic algebras, polyadic algebras, relation algebras.
- Representable and non-representable algebras.
- Basic properties of reduced products and ultraproducts.
- Connections with Ramsey Theory. Ultrafilter spaces.
- α-regular filters and their basic properties.
- α-universal models and Frayne's theorem.
- Ultrachains.
- α-saturated models.
- The Keisler-Shelah theorem.
- Ultraproducts and higher order formulas, ultratopologies.
- Applications: Non-standard Analysis. Stability Theory (Morely rank, stable models and theories, applications in Algebraic Geometry.

21) ELEMENTARY PRIME NUMBER THEORY

Course Coordinator: Antal Balog
Prerequisites: Basic analysis, Basic group theory

Book: M. B. Nathanson, Elementary methods in number theory, Springer, Graduate texts in mathematics 195, New York - Berling 2000, chapters 4, 6, 8, 9, 10.

Commitment: 3 hours/week, 3 credits

Contents:

- Elementary but more difficult methods in mutiplicative number theory. The elementary proof of the prime number theorem via Selberg's formula. Characters. The elementary proof of Dirichlet's theorem.
- Sieves: Brun's sieve (and Hooley's refinement), Selberg's sieve, the large sieve. Applications: upper estimate for twin primes.

22) COMBINATORIAL NUMBER THEORY

Course Coordinator: Imre Ruzsa
Prerequisites: Undergraduate number theory

Books:
1. H. Halberstam, K. F. Roth, K. F, Sequences, Clarendon, London, 1966; 2nd ed. Springer, 1983.
2. M. B. Nathanson, Additive number theory: Inverse problems and the geometry of sumsets, Springer, Graduate texts in mathematics 165, New York - Berlin 1996.

Commitment: 3 hours/week, 3 credits

Contents:

- Concepts of density. Estimates for the density of sumsets: the theorems of Schnirel-mann, Mann, Kneser, Erdős, Plünnecke. Inequalities for the cardinality of sumsets of finite sets of integers, residues, lattice points. Structure of sets with small sums.
- Sidon sets.

23) PROBABILISTIC METHODS IN NUMBER THEORY

Course Coordinator: Imre Ruzsa

Prerequisites: Basic probability theory
Books:
N. Alon, J. H. Spencer, P. Erdős, The probabilistic method, Wiley, New York, 1992.

Commitment: 3 hours/week, 3 credits

Contents:

- Tools from probability theory: Bernstein's inequality, the Esary-Proschan-Walkup (or Fortuin-Kasteleyn-Ginibre) inequality, Janson's inequality.
- Properties of random sets. Thresholds for properties: being a basis, being a Sidon set.
- Random constructions: regular bases, dense or regular Sidon sets, pseudo-squares with a dense sumset, essential components, sum-intersective sets.

24) PROBABILITY

Course Coordinator: Péter Major

Prerequisites: Introduction to Probability and Statistics I-II; Measure and integral.
Books:
1. L. Breiman: Probability. Addison-Wesley, Reading, Massachusetts (1968).
2. W. Feller: An Introduction to Probability Theory and its Applications, Vol. II. Second edition. Wiley, New York (1971).
3. A. Rényi: Probability Theory. North-Holland, Amsterdam (1970).
4. Y.S. Chow-H. Teicher: Probability Theory. Independence, Interchangeability, Martingales. Third edition. Springer, New York (1997).
5. P. Major: Series of Problems in Probability Theory. http://www.renyi.hu~major

Commitment: 3 hours/week, 3 credits

Contents:

Random variables, distributions, expectations, moments; Independence. Conditional distributions; Weak convergence of probability distributions, equivalent formulations; The method of characteristic functions in proving weak convergence: the central limit theorem; Laws of large numbers, large deviations, law of the iterated; logarithm. Elements of the theory of stochastic processes. Poisson process, Wiener process, Markov processes; Martingales.

25) MATHEMATICAL STATISTICS

Course Coordinator: Endre Csáki
Prerequisites: Stochastics, Probability, Measure and integral.
Books:
1. M.G. Kendall & A. Stuart: The Advanced Theory of Statistics. Vol. I., II. Griffin, London 1958, 1961.
2. C.R.Rao: Linear Statistical Inference and its Applications. Wiley, 1965.

Commitment: 3 hours/week, 3 credits

Contents:
General Concepts:
Statistical space, statistical sample. Important statistics. Empirical distribution. Histogram. Multidimensional normal distribution. Fisher information. Sufficiency. Completeness. Exponential family.
Theory of estimation:
Unbiased estimators. Efficiency. Cramer-Rao inequality. Admissibility. Asymptotic properties of estimators: consistency, asymptotic normality. Method of moments, least squares, maximum likelihood.
Theory of hypothesis testing.
Elements of Bayesian statistics.

26) INFORMATION THEORY

Course Coordinator: Imre Csiszár
Prerequisites: Introduction to probability and statistics, vector spaces over finite fields.
Books:
1. T.M. Cover & J.A. Thomas: Elements of Information Theory. Wiley, 1991.
2. I. Csiszar & J. Korner: Information Theory. Academic Press, 1981.

Commitment: 3 hours/week, 3 credits

Contents:
- Definition and formal properties of Shannon's information measures
- Source and channel models. Source coding, block and variable length codes, entropy rate. Arithmetic codes. The concept of universal coding.
- Channel coding (error correction), operational definition of channel capacity. The coding theorem for discrete memoryless channels. Shannon's source-channel transmission theorem.
- Outlook to multiuser and secrecy problems.
- Exponential error bounds for source and channel coding. Compound and arbitrary varying channels. Channels with continuous alphabets; capacity of the channel with additive Gaussian noise.
- Elements of algebraic coding theory; Hamming and Reed-Solomon codes.

27) INTRODUCTION TO THE THEORY OF COMPUTING

Course Coordinator: Vince Grolmusz
Prerequisites: -
Book:
T. H. Cormen, C. L. Leiserson and R. L. Rivest, Introduction to Algorithms, MIT Press, Cambridge, MA, 1990.

Commitment: 3 hours/week, 3 credits
Contents:
Communication games, examples. Dynamic programming: maximal interval-sum, largest all-one square submatrix, the optimal bracketing of matrix-products. The knapsack problem. The scaling method of Ibarra and Kim: approximating the optimum solution of the knapsack problem. Recursive functions. Halting problem. The domino-problem. Deterministic time- and space complexity classes. For any recursive f(x), there exists a recursive language, which is not in DTIME(f(x)).
Non-deterministic Turing-machines.
Other NP-complete problems: Hypergraph hitting-set, edge-cover, hypergraph 2-colorability. 3-chromatic graphs, Independent set is NP-complete. Subset-sum, Knapsack is NP-complete. Non-approximability results: graph-coloring. Parallel computing.

28) ALGORITHMS

Course Coordinator: Vince Grolmusz
Prerequisites: Undergraduate Algebra, Combinatorics, Advanced Calculus, and Theory of Computing.
Books:
1. T. H. Cormen, C. L. Leiserson and R. L. Rivest, Introduction to Algorithms, MIT Press, Cambridge, MA, 1990.
2. W.J. Cook, W.H. Cunningham, W.R. Pulleyblank, and A. Schrijver. Combinatorial Optimization. Wiley, 1998.

Commitment: 3 hours/week, 3 credits
Contents:
- Algebraic algorithms. Polynomials, FFT, Matrix algorithms. Number theoretical algorithms: prime searching, factoring, RSA cryptosystem.
- Sorting networks. Elementary parallel algorithms: MIN, sorting, graph algorithms on PRAMs. Determinant computing in parallel.
- Dynamic programming. Standard examples. Greedy algorithms. Matroids-an introduction. Graph algorithms.
- Combinatorial optimization an polyhedra. The basics of linear programming.
- Optimal matchings in bipartite and general graphs.
- Maximum flow problems. Minimum cuts in undirected graphs. Multicommodity flows. Minimum-cost flow problems.
- Outlook: Agorithms on the Web. Genetic algorithms.

29) COMPLEXITY THEORY

Course Coordinator: Vince Grolmusz

Prerequisite: Computing, Elementary Algebra.
Book: M. Sipser, Introduction to the Theory of Computation, PWS Publishing Company Boston, 1997.

Commitment: 3 hours/week, 3 credits

Contents:
- Formal models of computation: Turing machines, RAM machines.
- Reduction, complete languages for NP, P, NL, PSPACE. Savitch's theorem.
- Diagonal method: time- and space hierarchy.
- Randomization, randomized complexity classes, their relation to deterministic/ non-deterministic classes, examples.
- Communication complexity, deterministic, non-deterministic, relation to each other and to matrix rank.
- Decision trees: deterministic, non-deterministic, randomized, sensitivity of Boolean functions.

30) ERGODIC THEORY

Course Coordinator: Szász Domokos

Prerequisites: Measure Theory
Books:
1. I. P. Cornfeld - S. V. Fomin - Ya. G. Sinai: Ergodic Theory, Springer, 1982.
2. A. Katok - B. Hasselblatt: Introduction to the Modern Theory of Dynamical Systems, Cambridge Univ. Press, 1995.
3. K. Petersen: Ergodic theory, Cambridge University Press, 1989

Commitment: 3 hours/week, 3 credits

Contents:
- Dynamical systems and measure-preserving transformations
- Poincaré's recurrence theorem. Ergodic theorems of von Neumann and of Birkhoff and Khinchin. The notion of ergodicity
- Fourier methods for establishing ergodicity
- Symbolic dynamics, Bernoulli-shifts, topological Markov-chains
- Mixing, convergence to equilibrium
- Chaos and hyperbolicity (Hopf's method)
- Entropy theory (Kolmogorov-Sinai theorem), Shannon-McMillan-Breiman theorem
- Mixing property of the ideal gas
- Spectral theory

31) MATHEMATICAL METHODS IN STATISTICAL PHYSICS

Course Coordinator: Bálint Tóth
Prerequisites: Probability, Functional Analysis, Complex Functions.

Commitment: 3 hours/week, 3 credits

Contents:
- The object of study of statistical physics, basic notions.
- Curie-Weiss mean-field theory of the critical point. Anomalous fluctuations at the critical point.
- The Ising modell on Z^d.
- Analiticity I: Kirkwood-Salsburg equations.
- Analiticity II: Lee-Yang theory.
- Phase transition in the Ising model: Peierls' contour method.
- Models with continuous symmetry.

32) FRACTALS AND DYNAMICAL SYSTEMS

Course Coordinator: Károly Simon
Prerequisites: Ergodic Theory, Measure Theory

Books:
1. K. Falconer, Fractal geometry. Mathematical foundations and applications. John Wiley & Sons, Ltd., Chichester, 1990.
2. K. Falconer, Techniques in fractal geometry. John Wiley & Sons, Ltd., Chichester, 1997.
3. Y. Pesin, Dimension theory in dynamical systems. Contemporary views and applications Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1997.

Commitment: 3 hours/week, 3 credits

Contents:
- Fractal dimensions. Hausdorff and Packing measures.
- Basic examples of dynamically defined fractals. Horseshoe, solenoid.
- Young's theorem about dimension of invariant measure of a C^2 hyperbolic diffeomorphism of a surface.
- Some applications of Leddrapier- Young theorem.
- Barreira, Pesin, Schmeling Theorem about the local dimension of invariant measures.
- Geometric measure theoretic properties of SBR measure of some uniformly hyperbolic attractors.
- Solomyak Theorem about the absolute continuous infinite Bernoulli convolutions.

33) HIGHER LINEAR ALGEBRA

Course Coordinator: Pham Ngoc Anh
Prerequisites: Basic algebra
Books:
2. P.M. Cohn: Algebra, Vol. II, John Wiley, 1989.
4. P. Lancaster: Theory of Matrices, Academic Press, 1969.

Commitment: 3 hours/week, 3 credits
Contents:
- Structure of one endomorphism. Fitting's lemma, normal forms of matrices, Wedderburn's principal decomposition.
- Circulants, finite period, (sub-)groups of units and related invariants.
- Structure of two and more endomorphisms. Modules over free associative algebras and representation of quivers (directed graphs). Kronecker' classification.
- Witt's treatment of definite forms and reflection groups. Dynkin and Euclidean graphs.
- Witt's cancellation theorem. Witt rings.
- Tensor algebras, Clifford algebras, exterior algebras, Schubert cells.
- Spectral theory of matrices. Theorems of Gershgorin and Schur. Nonnegative matrices. The Perron-Frobenius theorem. Stochastic matrices; Markov chains.

34) REPRESENTATION THEORY I.

Course Coordinator: Mátyás Domokos
Prerequisites: Basic algebra , calculus, topology, the concept of manifolds
Books:
1. J. F. Adams, Lectures on Lie Groups, Benjamin, New York, 1969.
2. E. B. Vinberg, Linear Representations of Groups, Birkhauser, 1989.

Commitment: 3 hours/week, 3 credits

Contents:
This course is a first introduction to the subject for an audience with diverse mathematical interest.
- Basic concepts of group actions and linear representations, stabilizers, orbits, properties of completely reducible representations.
- Finite dimensional continuous representations of compact groups are unitary, the existence of Haar measure.
- Irreducible complex or real representations of Abelian groups, tensor product of representations, irreducible representations of direct products of groups.
- The Peter-Weyl Theorem on the matrix elements of irreducible representations of compact groups, decomposition of the regular representation.
- Physical applications of the character theory of finite groups.
- Special orthogonal group $S0_3$ and its double cover SU_2, their finite dimensional irreducible continuous representations. A treatment of the Laplace spherical functions.
- The concept of a Lie group and its Lie algebra, the connection between the representation theory of a Lie group and its Lie algebra.
- The representation theory of the symmetric group, Young tableaux, branching rules, Schur functions.

35) REPRESENTATION THEORY II.

Course Coordinator: Mátyás Domokos
Prerequisites: Representation Theory I.
Books:
1. W. Fulton and J. Harris, Representation Theory, Springer-Verlag, 1995.
2. I. G. Macdonald, Symmetric Functions and Hall polynomials, Clarendon Press, Oxford, 1995.

Commitment: 3 hours/week, 3 credits
Contents:
Schur-Weyl duality, polynomial representations of the general linear group, Schur functors, plethysms.
- Representations of the classical Lie algebras and the corresponding Lie groups.
- Representation theory of the Lorentz group.
- Maximal tori, dominant characters, weight lattices, highest weight of an irreducible representation, Weyl's character formula.
- Correspondence of compact Lie groups and reductive algebraic groups, complete reducibility of finite dimensional linear representations.
- Induced representations, Frobenius reciprocity.
- Universal enveloping algebra, Casimir operators, Verma modules.
- Schur algebras, and other connections to the representation theory of finite dimensional associative algebras.

36) UNIVERSAL ALGEBRA AND CATEGORY THEORY

Course Coordinator: László Márki
Prerequisites: -
Books:
1. S. Burris - H. P. Sankappanavar: A Course in Universal Algebra, Springer, 1981.
2. G. Gratzer: Universal Algebra, 2nd ed., Springer, 1979
3. J. Almeida: Finite Semigroups and Universal Algebra, World Scientific, 1994.

Commitment: 3 hours/week, 3 credits

Contents:
- Algebra, many-sorted algebra, related structures (subalgebra lattice, congruence lattice, automorphism group, endomorphism monoid), factor algebras, homomorphism theorem, subdirect product, subdirectly irreducible and simple algebras, Birkhoff's theorem, Gratzer--Schmidt theorem, ultraproduct, Los lemma.
- Variety, word algebra, free algebras, identities, Birkhoff's theorem, pseudovariety, implicit operation, pseudoidentity, Reiterman's theorem, equational implication, quasivariety, Kogalovskii's theorem, fully invariant coungruence, Birkhoff's completeness theorem, Malcev type theorems.
- Category, functor, natural transformation, special morphisms. Duality, contravariance, opposite, product of categories, comma categories. Universal arrow, Yoneda lemma, coproducts and colimits, products and limits, complete categories, groups in categories.

Adjoints with examples, reflective subcategory, equivalence of categories, adjoint

37) TOPICS IN GROUP THEORY

Course Coordinator: Péter Pál Pálfy
Prerequisites: Basic algebra 1-3
Books:
1. M. Aschbacher, Finite group theory, Cambridge Univ. Press, 1986.
2. M. Suzuki, Group theory I-II, Springer, 1986.

Commitment: 3 hours/week, 3 credits
Contents:
- Free groups and presentations (Nielsen--Schreier Theorem, residual finiteness of free groups, varieties of groups)
- Commutator calculus (Three-Subgroup-Lemma, lower central series, commutator collecting process)
- The Burnside problem (solution for small exponents, the state of art for large exponents)
- Abelian groups
- Groups of prime-power order
- Nilpotent and supersolvable groups
- Finite solvable groups (Hall subgroups, Sylow systems, formations)
- The transfer homomorphism (Burnside's normal p-complement theorem, Grun's Theorems, Frobenius' Criterion)
- Classical linear groups (simplicity of PSL(n,F))
- Finite simple groups (the list of simple groups, Mathieu groups)
- Frobenius groups (induced characters, Frobenius kernel)
- Group extensions (Schur--Zassenhaus Theorem)
- Subgroup lattices (Theorems of Ore and Iwasawa)

38) TOPICS IN RING THEORY I.

Course Coordinator: Pham Ngoc Anh
Prerequisites: Basic algebra 1-3
Books:
1. I. Kaplansky: Fields and Rings, The University of Chicago Press, 1972.
2. Lam: A First Course in Noncommutative Rings, Springer, 1991.

Commitment: 3 hours/week, 3 credits

Contents:
- Matrix rings, rings associated to directed graphs, skew polynomial rings, skew Laurent and power series rings, skew group rings, enveloping algebras of Lie algebras, Weyl algebras, free associative algebras, tensor .
- Jacobson theory, rings of endomorphisms of vector spaces, Burnside and Kurosh problems, simple nil rings and Kőthe's problem
- Categorical module theory I: generators and cogenerators, flat modules and characterization of regular rings, Bass' theory of (semi-)perfect rings, Bj\"ork's results, examples on rings with the descending chain condition on finitely generated one-sided ideals
- Categorical module theory II: Morita theory on equivalence and duality, projective generators and injective cogenerators, Pickard groups

39) TOPICS IN RING THEORY II.

Course Coordinator: Anh Pham Ngoc
Prerequisites: Basic Algebra 1-3
Books:
1. C. Faith: Algebra II: Ring Theory, Springer-Verlag, 1991.
2. T. Y. Lam: A First Course in Noncommutative Rings, Springer, 1991.

Commitment: 3 hours/week, 3 credits

Contents:
- Goldie's theory
- Noncommutative localization, quotient constructions, Artin's problems on division rings
- Separable algebras, principal Wedderburn theorem, central simple algebras, cyclic (division) algebras, p-algebras, involution of algebras,
- Auslander's treatment of first Brauer conjecture for artin algebras, results on Krull-Schmidt theorem
- Frobenius and quasi-Frobenius rings, serial rings

40) PERMUTATION GROUPS

Course Coordinator: László Pyper and Péter Pál Pálfy
Prerequisites: Undegraduate group theory

Books:
1. P. J. Cameron: Permutation Groups, Cambridge Univ. Press, 2001.
2. J.D. Dixon & B. Mortimer: Permutation Groups, Springer, 1996

Commitment: 3 hours/week, 3 credits

Contents:

- Orbits and transitivity
- The orbit-counting lemma and its consequences
- Extensions; Kantor's lemma
- Blocks and primitivity
- Wreath products
- Doubly transitive groups: examples
- Burnside's theorem on normal subgroups of doubly transitive groups
- Further construction of permutation groups
- Consequences of CFSG: Cameron's theorem; classification of doubly transitive groups; rank 3 permutation groups
- Jordan groups
- Finitary permutation groups
- Oligomorphic groups

41) LIE GROUPS AND ALGEBRAS

Course Coordinator: Péter Pál Pálfy
Prerequisites: Basic algebra 1-3, Representation Theory I.
Books:
1. M. Postnikov, Lie groups and Lie algebras, Mir, Moscow, 1986.
2. J. P. Serre, Lie algebras and Lie groups, 2nd ed., Springer, 1992.

Commitment: 3 hours/week, 3 credits

Contents:
- Solvable and nilpotent Lie algebras, Engel's Theorem, Lie's Theorem Trace forms, Cartan's Criterion
- Semisimple Lie algebras, Cartan subalgebras, root spaces
- Root systems, Weyl groups, Dynkin diagrams, classification, Chevalley basis
- Universal enveloping algebra, Poincar\'e--Birkhoff--Witt Theorem, free Lie algebras
- Exponential map, Dynkin polynomials, Campbell--Hausdorff formula
- Canonical coordinates, local Lie subgroups
- Coverings, universal covering groups
- Lie subgroups, factor groups and factor spaces
- Ado's Theorem

42) COMMUTATIVE ALGEBRA

Course coordinator: Tamás Szamuely
Prerequisites: Basic Algebra 1-3, Homological Algebra
Books:
1. M. F. Atiyah, I. G. MacDonald: Introduction to Commutative Algebra. Addison-Wesley, 1969.
2. H. Matsumura: Commutative Ring Theory. Cambridge University Press, 1988.
3. D. Eisenbud: Commutative Algebra with a View Toward Algebraic Geometry. Springer, 1995.
4. J.-P. Serre: Local Algebra. Multiplicities. Springer, 1999.

Commitment: 3 hours/week, 3 credits

Contents:
- Review of basic concepts about commutative rings. Chain conditions, Noetherian and Artinian rings and modules. Associated primes and primary decomposition.
- The prime spectrum of a ring, localisation.
- Basic dimension theory. The Krull dimension of a finite dimensional algebra over a ring.
- Integral extensions, integral closure. Structure of discrete valuation rings and Dedekind domains. Example: rings of algebraic integers in number fields and their extensions.
- Completion of a local ring, the associated graded ring.
- Application of methods of homological algebra to local rings.

43) ALGEBRAIC NUMBER THEORY

Course coordinator: Tamás Szamuely
Prerequisites: Basic Algebra 1-3
Books:
1. J.-P. Serre: A Course in Arithmetic. Springer, 1970.
2. S. Lang: Algebraic Number Theory. Springer, 1986.

Commitment: 3 hours/week, 3 credits
Contents:
- Ideal theory in Dedekind domains: decomposition as a product of prime ideals, the class group.
- Behaviour of prime ideals in field extensions: decomposition and ramification. Localisation and completion: local fields, Hensel's lemma. Structure of unramified, totally ramified and tamely ramified extensions. Ramification theory in Galois extensions, decomposition and inertia groups, henselisation. Structure of the absolute Galois group of a local field.
- Minkowski's theory of lattice points in convex bodies.
- Basic finiteness theorems for number fields: finiteness of the class group, structure of the group of units. Minkowski's bound for discriminants, arithmetic monodromy.
- Examples: quadratic and cyclotomic fields.
- Analogies with algebraic curves. Applications: diophantine equations, abelian groups as Galois groups over $\bf Q$.
- Quadratic forms: review of algebraic theory. Quadratic forms over finite and local fields, Hasse-Minkowski theorem. Classification of quadratic forms over $\bf Q$.
- Zeta and L-functions of number fields, density theorems of Dirichlet and Chebotarev.

44) GEOMETRIC GROUP THEORY

Course Coordinators: Gabor Elek, Gabor Moussong
Prerequisites: Undergraduate group theory. Real analysis.
Books:
P. de la Harpe: Topics in Geometric Group Theory, Univ. of Chicago Press, 2000.

Commitment: 3 hours/week, 3 credits

Contents:
- Free groups, free products and amalgams. Group actions on trees.
- Finitely generated groups, volume growth, Cayley graphs, quasi- isometries, ends, boundaries and their invariance.
- Finitely presented groups. Rips complexes.
- Amenable groups.
- Nilpotent groups. Gromov's theorem on polynomial growth.
- Groups with Kazhdan's property (T). The expander problem.
- Bloch-Weinberger homologies and their applications.
- Hyperbolic groups. Gromov's boundary.
- Bounded harmonic functions on graphs and groups.

45) RESIDUALLY FINITE GROUPS

Course Coordinator: László Pyber

Prerequisites: Undergraduate Group Theory

Books: W. Magnus: Residually finite groups (survey) and related papers.

Commitment: 3 hours/week, 3 credits

Contents:

- Residual properties of free groups; the theorems of Iwasawa and Katz, Magnus, Wiegold
- The theorem of G. A. Jones on proper group-varieties, the Magnus conjecture
- Residual properties of free products
- Polycyclic groups are residually finite
- Linear groups are residually finite
- Basic properties of residually finite groups, the solvability of the word, problem, hopficity
- The automorphism group of a residually finite group is residually finite
- Conjugacy separability and LERF groups
- The restricted Burnside problem; Hall-Higman reduction
- Grigorchuk groups
- Residually finite groups of finite rank
- Profinite completions
- Every abstract subgroup of finite index in a finitely generated pro-p, group is open; Serre's problem
- Subgroup growth of free groups and nilpotent groups
- Groups of intermediate subgroup growth

46) INVARIANT THEORY

Course Coordinator: Mátyás Domokos

Prerequisites: Basic Algebraic Geometry, Commutative Algebra

Books:

1. R. Goodman, N. R. Wallach, Representations and Invariants of Classical Groups, Encyclopedia of Math. Its Appl. 68, Cmbridge Univ. Press, Cambridge, 1998.
2. H. Kraft, Geometrische Methoden in der Invariantentheorie, Vieweg,\linebreak Braunschweig/Wiesbaden, 1985.

Commitment: 3 hours/week, 3 credits

Contents:

- The First and Second Fundamental Theorems for the invariants of classical groups acting on vectors and covectors
- Generators of polynomial invariants of finite groups. Finite generation of algebras of invariants of reductive groups
- The null-cone
- Affine quotients
- Rational invariants
- Coarse and fine moduli spaces, linearization of an action, projective quotients
- The algebras of invariants of reductive groups, Hilbert series, Gorenstein property
- The Sheppard-Todd-Chevalley Theorem on invariants of finite pseudo-reflection groups.

47) SEMIGROUP THEORY

Course Coordinator: László Márki
Prerequisites: Basic Algebra 1, Universal Algebra and Categories
Books:

1. J. M. Howie: Fundamentals of Semigroup Theory, Oxford University Press, 1995.
2. J. E. Pin: Varieties of Formal Languages, North Oxford Academic, 1986.
3. M. V. Lawson: Inverse Semigroups, World Scientific, 1998.
4. P. A. Grillet: Semigroups, Marcel Dekker, 1995.

Commitment: 3 hours/week, 3 credits

Contents:

- Basic notions, semigroups of transformations, free semigroups, Green's equivalences, regular D-classes, regular semigroups
- (0-)Simple semigroups, principal factors, completely (0-)simple semigroups
- Languages, syntactic monoids, pseudovarieties
- Inverse semigroups, elementary properties, fundamental inverse semigroups, free inverse semigroups
- Commutative semigroups, archimedean decomposition, finitely generated commutative semigroups

48) BASIC ALGEBRAIC GEOMETRY

Course coordinator: Károly Böröczky, Jr.
Prerequisites: Commutative algebra course

Books:

1. I.R. Shafarevich: Basic Algebraic Geometry I. Springer, 1994.
2. D. Mumford: Algebraic Geometry I: Complex Projective Varieties. Springer, 1976.
3. R. Hartshorne: Algebraic Geometry. Springer, 1977.

Commitment: 3 hours/week, 3 credits

Contents:

- Quasi-projective varieties over an algebraically closed field, morphisms, rational maps. Products, dimension and other basic properties.
- Local theory: tangent cones, differentials, smooth and normal points, branch locus of a map. Normalization of a variety, desingularization of a curve.
- Projective techniques: projections, fibrations, Bertini theorems. Blow ups of points and subvarieties.
- Basic intersection theory on a surface, Hodge index theorem, Bezout's theorem. Factorization of birational maps of surfaces, embedded resolution for curves.

49) THE LANGUAGE OF SCHEMES

Course coordinator: Tamás Szamuely
Prerequisites: Commutative algebra and Homological algebra courses. Basic Algebraic Geometry is strongly recommended.
Books:
1. R. Hartshorne: Algebraic Geometry. Springer, 1977.
2. D. Mumford: The Red Book of Varieties and Schemes. Springer, 1999.
3. D. Eisenbud, J. Harris, Geometry of Schemes. Springer, 1999.

Commitment: 3 hours/week, 3 credits

Contents:
- Basic definitions and properties: abelian sheaves on a topological space, the category of schemes.
- Quasi-coherent and coherent sheaves, line bundles. Projective schemes, the Serre correspondence for quasi-coherent modules.
- Cohomology of coherent sheaves: cohomological dimension, Serre's vanishing theorem, finiteness of the cohomology of projective schemes.
- Castelnuovo-Mumford regularity.
- Construction of schemes from their functor of points: Grassmannians, Hilbert schemes.
- Applications of Hilbert schemes (survey): Picard schemes, Jacobians, moduli problems.

50) GALOIS GROUPS IN GEOMETRY

Course coordinator: Tamás Szamuely
Prerequisites: Commutative Algebra, Galois Theory, basic topology and complex function theory
Books: Handouts

Commitment: 3 hours/week, 3 credits

Contents:
- Galois theory of fields (review). Infinite Galois extensions, Krull topology. Reformulation of Galois theory in terms of \'etale algebras.
- Cover(ing space)s in topology. Galois covers and group actions. The main theorem of Galois theory for covers, relation to the fundamental group. Construction of the universal cover. Locally constant sheaves and their homotopy classification, application to differential equations.
- Riemann surfaces, branched covers. The fundamental group of the punctured line.

Basic properties of schemes. The algebraic fundamental group of a locally Noetherian scheme, relation with the topological theory. Galois groups as fundamental groups, homotopy exact sequence. Fundamental groups of Dedekind schemes, relation with Galois theory. The fundamental group of the projective line minus three points, Belyi's theorem

51) ALGEBRAIC CURVES AND JACOBIAN VARIETIES

Course Coordinator: Tamás Szamuely
Prerequisites: Language of Schemes course, Homological Algebra course.
Books:
1. R. Hartshorne: Algebraic Geometry. Springer, 1977.
2. G. Cornell, J. H. Silverman (eds.) Arithmetic Geometry. Springer, 1986.
3. J.-P. Serre: Algebraic Groups and Class Fields, Springer, 1988.

Commitment: 3 hours/week, 3 credits

Contents:

- Basic theory of algebraic curves: divisors, differentials, Riemann-Roch formula and applications. Covers of algebraic curves, the Hurwitz formula. Relation with the transcendental theory.
- Algebraic curves over finite fields, Weil's theorem on the ``Riemann Hypothesis''. Bounds on the number of rational points.
- The construction of Jacobian varieties. Applications (partly only surveyed): abelian covers of curves, geometric class field theory, Mordell's conjecture, an-abelian geometry.

52) THE ARITHMETIC OF ELLIPTIC CURVES

Course Coordinator: Tamás Szamuely
Prerequisites: Basic Algebraic Geometry, Algebraic Number Theory.
Books:
1. J. H. Silverman: The Arithmetic of Elliptic Curves. Springer, 1986.

Commitment: 3 hours/week, 3 credits

Contents:

- Geometry of elliptic curves: Weierstrass equations, group law, j-invariant, singular curves and degenerate laws.
- The structure of the group of points of an elliptic curve over a p-adic field, good and bad reduction.
- Rudiments of Galois cohomology, weak Mordell-Weil theorem.
- Heights on projective space over a global field, strong Mordell-Weil theorem.
- Torsors over an elliptic curve, Selmer and Tate-Shafarevich groups, obstructions to the Hasse principle.
- Elliptic curves over finite fields, “Riemann Hypothesis”'.
- Advanced topics (optional): moduli of elliptic curves, semi-stable reduction, Weil curves, application to Fermat's Last Theorem (survey).

53) HODGE THEORY

Course Coordinator: Endre Szabó
Prerequisites: Basic Algebraic Geometry, Basic Differential Geometry, Functional Analysis, Algebraic Topology and Homological Algebra
Books:

1. P. Griffiths and J. Harris: Principles of algebraic geometry, Wiley Classic Library, 1994.
2. R. O. Wells: Differential analysis on complex manifolds, Graduate texts in mathematics, Springer-Verlag, 1980.

Commitment: 3 hours/week, 3 credits

Contents:

- Review of complex manifolds, metrics, connections and curvature, respectively of Hodge * operator and Laplace operator;
- Kahler manifolds, DeRham cohomology, Dolbeault cohomology; Harmonic forms, Hodge theorem, Serre duality, Kunneth formula, Hodge decomposition, Lefschetz decomposition;
- Intersection form and polarization properties, the Hodge-Riemann bilinear relations;
- Kodaira vanishing theorem, Kodaira embedding theorem;
- Lefschetz theorem on hyperplane sections;
- Hodge conjecture, Lefschetz theorem on (1,1) classes;
- Algebraic DeRham complex, differential forms with logarithmic singularities.

54) INTRODUCTION TO CLASSIFICATION THEORY

Course Coordinator: Endre Szabó
Prerequisites: Basic Algebraic Geometry, The Language of Schemes.
Books:

1. H. Clemens, J. Kollar and S. Mori, Higher Dimensional Complex Geometry, Astérisque vol. 166, Soc. Math. France, 1988.
2. R. Hartshorne: Algebraic Geometry, Springer 1977.

Commitment: 3 hours/week, 3 credits

Contents:

- Classification of curves, birational maps of surfaces - an overview;
- Minimal surfaces, classification of surfaces, canonical surfaces;
- Minimal model conjecture, Abundance conjecture;
- Cone of curves, Kleiman's criterion for ampleness, Cone theorem, Contraction theorem;
- Flips, Flops, Flip conjecture, Minimal Model Program;
- Singularities, resolutions, discrepancy;
- Log Minimal Model Program;
- Classification of 3 dimensional terminal singularities;
- Minimal Model Program in 3 dimensions;
- Applications

55) TORIC VARIETIES

Course Coordinator: Károly Böröczky, Jr.
Prerequisites: Linear Algebra, Basic Algebraic Geometry
Books:
W. Fulton: Introduction to toric varieties. Princeton University Press, Princeton, NJ, 1993.

Commitment: 3 hours/week, 3 credits

Contents:

- Rational cones and Fans.
- Affine toric varieties, toric varieties, characterization of the projective and the complete toric varieties.
- The moment map.
- Resolution of singularities in toric setting.
- Invariant line bundles.
- Toric singularities.
- Intersection numbers.
- The Chow ring of a toric variety.
- Counting lattice points and the Hirzebuch-Riemann-Roch formula.
- About the coefficients of the Ehrhart formula.
- The Alexandrov-Fenchel inequality and the Hodge intersection inequality.
- Some applications of toric varieties to mirror symmetry.

56) DYNAMICAL SYSTEMS

Course Coordinator: Szász Domokos
Prerequisites: Ergodic Theory
Books:

1. I.P. Cornfeld and S. V. Fomin and Ya. G. Sinai, Ergodic Theory, Springer, 1982
2. A. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems, Cambridge Univ. Press, 1995

Commitment: 3 hours/week, 3 credits

Contents:

- Kesten-Furstenberg theorem, Kingman's subadditive ergodic theorem.
- Oseledec' multiplicative ergodic theorem, Lyapunov exponents.
- Thermodynamic formalism, Markov-partitions.
- Chaotic maps of the interval, expanding maps, Markov-maps.
- Chaotic conservatice systems. The ergodic hyothesis. Billiards, hard ball systems. The standard map.
- Chaotic non-conservative (dissipative) systems. Strange attractors. Fractals. Exponenets and dimensions. Map of the solenoid.
- Stability: invariant tori and the Kolmogorov-Arnold-Moser theorem.
- Anosov-maps. Invariant manifolds. SRB-measure.

57) APPROXIMATION THEORY

Course Coordinator: András Kroó
Prerequisites: Real and Functional Analysis
Books:
R. DeVore and G. Lorentz, Constructive Approximation, Springer, 1991.

Commitment: 3 hours/week, 3 credits

Contents:

Stone-Weierstrass theorem, positive linear operators, Korovkin theorem. Best Approximation (Haar theorem, Chebyshev polynomials, Best approximation in different norms). Polynomial inequalities (Bernstein, Markov, Remez inequalities). Splines (B-splines, Euler and Bernoulli splines, Kolmogorov-Landau inequality). Direct and connverse theorems of best approximation (Favard, Jackson, and Stechkin Theorems). Approximation by linear operators (Fourier series, Fejér operators, Bernstein polynomials). Müntz theorem.

58) PARTIAL DIFFERENTIAL EQUATIONS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Undergraduate Calculus, Linear Algebra, Real and Complex Analysis

Books:
1. L.C. Evans, Partial Differential Equations, Graduate Studies in Math. 19, AMS, Providence, Rhode Island.
2. A. Friedman, Partial Differential Equations, Holt, Rinehart and Winston, Inc.

Commitment: 3 hours/week, 3 credits

Contents:

- Physical models involving linear and nonlinear partial differential equations
- Elliptic equations. Maximum principles
- Variational solutions for elliptic boundary value problems
- Parabolic equations
- Hyperbolic equations and systems.
- Theory for nonlinear partial differential equations. Variational and nonvariational techniques.
- Conservation laws
- Applications to specific problems

59) NONLINEAR EVOLUTION EQUATIONS AND APPLICATIONS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Real and Complex Analysis, Functional Analysis
Books:

1. H. Brezis, Operateurs maximaux monotones et semigroupes de contractions dans les espaces de Hilbert, North Holland, Amsterdam, 1973.
2. G. Morosanu, Nonlinear Evolution Equations and Applications, Reidel, 1988.

Commitment: 3 hours/week, 3 credits

Contents:

- Preliminaries of linear and nonlinear functional analysis
- Existence and regularity of solutions to evolution equations in Hilbert spaces
- Boundedness of solutions
- Stability of solutions. Strong and weak convergence results. Periodic forcing. The asymptotic dosing problem
- Applications: Infinite delay equations, nonlinear parabolic problems, hyperbolic systems. Specific examples in viscoelasticity, heat conduction theory, electrical and electronic engineering, hydraulics, a.o.

60) FUNCTIONAL METHODS IN DIFFERENTIAL EQUATIONS

Course Coordinator: Gheorghe Morosanu

Prerequisites: Real and Complex Analysis, Functional Analysis
Books:

1. V.-M. Hokkanen and G. Morosanu, Functional Methods in Differential Equations, Chapman & Hall/CRC, 2002.
2. A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer-Verlag, 1983.

Commitment: 3 hours/week, 3 credits

Contents:

- Preliminaries of linear and nonlinear functional analysis
- Degenerate and non-degenerate elliptic boundary value problems. Applications to capillarity theory
- Nonlinear parabolic problems with algebraic or differential boundary conditions. New models in heat propagation theory and diffusion phenomena.
- Hyperbolic problems with algebraic and differential boundary conditions. New models in engineering (unsteady fluid flow with nonlinear pipe friction, electrical transmission phenomena, etc.)
- The abstract Fourier method. Applications in acoustics.
- The semigroup approach and applications to PDE.

61) COMPLEX MANIFOLDS

Course Coordinator: Róbert Szőke
Prerequisites: Basic topology, real analysis in several variables, functional analysis and complex analysis in one variable.
Books:
1. K. Kodaira: Complex manifolds, Holt, 1971.
2. R.O. Wells: Differential analysis on complex manifolds, Springer, 1979.

Commitment: 3 hours/week, 3 credits

Contents:
- Basic definitions, examples and constructions.
- Differential forms on manifolds, (p,q) forms. Tangent bundle, vector bundles, bundle valued forms and Dolbeault cohomology groups, metrics, Hodge * operator.
- Sobolev spaces of sections, differential operators between vector bundles and their adjoint, symbol. Pseudo-differential operators.
- Parametrix for elliptic differential operators, fundamental decomposition theorem for self-adjoint elliptic operators and complexes. Harmonic forms, complex Laplacian, Kahler manifolds, Hodge decomposition theorem on compact Kahler manifolds.

62) GEOMETRIC ANALYSIS

Course Coordinator: Jerry L. Kazdan
Prerequisites: Students will be assumed to know basic analysis---but all the background in PDE and geometry will be given as needed in the course.
Books:
1. Evans, Lawrence C., "Partial Differential Equations", in the series Graduate studies in mathematics, v. 19, American Math. Society, 1998.
2. Robert Hardt and Michael Wolf (editors), "Nonlinear Partial Differential Equations in Differential Geometry" - AMS, 1996, 339 pp.
3. Kazdan, Jerry L. "Lecture Notes on Applications of Partial Differential Equations to Some Problems in Differential Geometry", (available from the Internet at: http://www.math.upenn.edu/~kazdan/)
4. Li, Peter, "Lecture Notes on Geometric Analysis" (available from the Internet at: http://www.math.uci.edu/faculty/pli.html)

Commitment: 3 hours/week, 3 credits

Contents:
Some applications of analysis, particularly partial differential equations (PDE), to some problems in geometry. For instance, we will prove the Hodge decomposition theorem, Newlander-Nirenberg theorem, the spectrum of the Laplacian, as well as some applications involving nonlinear PDE's arising in Riemannian geometry. The course will be flexible enough to include some topics that the class would like to cover.

63) BLOCK DESIGNS

Course Coordinator: Tamás Szőnyi
Prerequisites: Some introductory course to discrete mathematics, e.g. Introduction to graph theory and combinatorics
Books:
E.F. Assmuss, J.D. Key: Designs and their codes, Cambridge University Press, 1992.

Commitment: 3 hours/week, 3 credits

Contents:

- The theorem of de Bruijn-Erdős and some generalizations. Projective planes and spaces. Block designs, square (or: symmetric) designs, Fisher's inequality. Non-existence results for square designs: the theorem of Bruck-Ryser-Chowla.
- Recursive constructions: derived design, residual design. Steiner triple systems. The constructions of Kirkman and Skolem. Hadamard matrices, Hadamard designs, biplanes. *t*-designs. Extension of a design: Cameron's theorem on the extension of square designs. Asymptotic results for block designs: the theorem of Wilson and Teirlinck.
- Strongly regular graphs, applications of eigenvalues. Absolute bound, Krein-conditions. Uniqueness of certain strongly regular graphs. Block designs and strongly regular graphs. Quasiresidual block designs, the theorem of Hall-Connor.
- Collineations and polarities in block designs. Difference sets. Singer's theorem, the multiplier theorems by M. Hall and their applications.
- Elements of coding theory. Linear codes, perfect codes.
- Construction of the Golay codes and Witt designs. Mathieu groups.

64) Hypergraphs, Set Systems, Intersection Theorems

Course Coordinator: Gyula Katona
Prerequisites: basic notions of combinatorics (graphs), linear algebra (finite fields) and probability theory (Tchebiseff's inequality)
Books:

1. L. Lovász, Combinatorial Problems and Exercises, 2nd edition, Akadémiai Kiadó, Budapest, 1993
2. K. Engel, Sperner theory, Cambridge University Press, 1997.
3. N. Alon, J.H. Spencer, The Probabilistic Method, John Wiley & Sons, 1992.
4. J. Matousek, Geometric Discrepancy, Algorithms and Combinatorics 18, Springer-Verlag, 1999.
5. B. Bollobás, Combinatorics, Set Systems, Hypergraphs, Families of Vectors & Probabilistic Combinatorics Combinatorics, Paperback, Cambridge University Press, 1986.

Commitment: 3 hours/week, 3 credits

Contents:

SPERNER THEORY AND POSETS:

- Sperner's proof, YBLM inequality (Lubell's, Bollobas' proof) extremal families (Füredi's proof)

Chain decomposition (Erdős' theorem)

- Related results, intersecting Sperner, k-Sperner (Erdős)
- Posets, Dilworth theorem
- Sum of vectors, Littlewood Offord (Kleitman's theorem)

DISCRETE ISOPERIMETRIC PROBLEMS:

- Harper's theorem (Frankl-Füredi's proof)
- Kruskal-Katona (Frankl's proof), Lovász' version
- Shadows for intersecting, Katona (for ℓ-intersecting systems)
- q^n, Hart's thm, A. Moon about |A||B|

INTERSECTION THEOREMS:

- Erdős-Ko-Rado theorem (left shifting)
- Ahlswede-Khachatrian theorem (left shifting again)
- Non-uniform t-wise l-intersecting families (results of Frankl)
- (A_i B_i) Bollobás theorem, Lovász's proof, Frankl's results
- Delta systems (Erdős-Rado theorem), kernel (Calcyinska-Carlowitz, Erdős-Lovász)
- Hajnal's theorem, kernels Erdős-Ko-Rado theorem from Kruskal-Katona theorem t-wise intersecting families
- Algebraic methods (Ray-Chaudhuri-Wilson, Frankl-Wilson)
- m (n,k,L) (Ryser) Frankl-Rödl theorem
- Structure intersection theorems (Chung, Frankl, Graham)
- Borsuk's conjecture (an application)

CODES AND DESIGNS

- Kneser graphs (Lovász-Bárány, Alon) generalizations (Frankl, Frankl-Füredi)
- Baranyai's theorem
- Discrepancy, B-property
- Packing and covering (Frankl-Rödl-Pippenger-Spencer)
- Tolhuizen's theorem
- Mastermind (Lindstrom, Chvatal)
- Vapnik-Chervonekis dimension
- Superimposed codes

65) SELECTED TOPICS IN GRAPH THEORY

Course Coordinator: Vera T. Sós
Prerequisites: -
Commitment: 3 hours/week, 3 credits

Contents:
The subject of this course changes from time to time depending on the fields of interest of students.

66) FINITE PACKING AND COVERING

Course Coordinator: Károly Böröczky, Jr.
Books: handouts
Commitment: 3 hours/week, 3 credits
Contents:
Planar arrangements: Translative packings of a centrally symmetric convex domain, the Oler inequality. Translative coverings by a centrally symmetric convex domain, the Fejes Tóth inequality. The optimal packing of equal Euclidean circles (G. Wegner). Density inside r-convex domains for arrangements of equal circles in the hyperbolic plane. The extremal perimeter for packings and coverings by congruent convex domains. The maximal perimeter for coverings by equal Euclidean circles. The Hadwiger number in the plane. Clouds in the plane.
Higher dimensional arrangements: Optimal arrangements of balls in the spherical space. The Sausage Conjecture and Theorem for Euclidean ball packings. The extremal mean width for packings and coverings by congruent convex bodies. The Hadwiger number in high dimensions. Clouds in high dimensions. Parametric density for translative arrangements. The Wulff shape for translative lattice packings.

67) PACKING AND COVERING

Course Coordinator: Gábor Fejes Tóth
Prerequisites: Geometry, Basic Linear Algebra
Books:

1. L. Fejes Tóth: Regular figures, Pergamon Press, 1964.
2. J. Pach and P.K. Agarwal: Combinatorial geometry, Academic Press, 1995.
3. C.A. Rogers: Packing and covering, Cambridge University Press, 1964.

Commitment: 3 hours/week, 3 credits
Contents:
Theorem of Groemer concerning the existence of densest packings and thinnest coverings. Dirichlet cells, Delone triangles. Theorems of Thue and Kershner concerning densest circle packings and thinnest circle coverings. Packing and covering of incongruent circles. Theorems of Dowker, generalized Dirichlet cells. Packing and covering of congruent convex discs: theorems of C.A. Rogers and L. Fejes Tóth.
The moment theorem. Isoperimetric problems for packings and coverings. Existence of dense packings and thin coverings in the plane: p-hexagons, extensive parallelograms, theorems of W. Kuperberg, D. Ismailescu, G. Kuperberg and W. Kuperberg. The theorem of E. Sas. Multiple packing and covering of circles. The problem of Thammes; packing and covering of caps on the 2-sphere. The moment theorem on S^2, volume estimates for polytopes containing the unit ball. Theorem of Lindelöff, isoperimetric problem for polytopes. Packing and covering in the hyperbolic plane.
Packing of balls in E^d the method of Blichfeldt, Rogers' simplex bound. Packing in S^d, the linear programming bound. Theorem of Kabatjanskii and Levenstein. Covering with balls in E^d the simplex bound of Coxeter, Few and Rogers. Existence of dense lattice packings of symmetric convex bodies: the theorem of Minkowski-Hlawka. Packing of convex bodies, difference body, the theorem of Rogers and Shephard concerning the volume of the difference body. Construction of dense packings via codes. The theorem of Rogers concerning the existence of thin coverings with convex bodies. Approximation of convex bodies by generalized cylinders, existence of thin lattice coverings with convex bodies

68) CONVEX POLYTOPES

Course Coordinator: Károly Böröczky, Jr.

Prerequisites: Basic linear algebra

Books:
G.M. Ziegler: Lectures on polytopes. Springer, 1995.

Commitment: 3 hours/week, 3 credits

Contents:

Rationale: Introducing the basic combinatorial properties of convex polytopes. Polytopes as convex hull of finite point sets or intersections of halfspaces. Faces of polytopes. Examples: Simplicial, simple, cyclic and neighbourly polytopes. Polarity for polytopes. The Balinski theorem. Discussion of the Steinitz theorem for three polytopes. Realizability using rational coordinates. Gale transform and polytopes with few vertices. The oriented matroid of a polytope Shelling Euler-Poincaré formula h-vector of a simplicial polytope, Dehn-Sommerfield equations Upper bound theorem Stresses Lower bound theorem Weight algebra Sketch of the proof of the g-theorem.

69) COMBINATORIAL GEOMETRY

Course Coordinator: János Pach

Prerequisites: Geometry, Basic linear algebra, Analysis, Discrete mathematics

Books:
J. Pach and P. Agarwal: Combinatorial Geometry, Wiley, New York, 1995

Commitment: 3 hours/week, 3 credits

Contents:

- Extremal graph theory
- Repeated distances among points
- Arrangement of lines
- Applications of the bounds on incidences
- Geometric graphs
- Epsilon nets and transversals of hypergraphs
- Geometric discrepancy
- Various topics from combinatorial geometry: theorems of Erdős-Szekes
- Gallai-Sylvester, Kuratowsky

70) GEOMETRY OF NUMBERS

Course Coordinator: András Bezdek
Prerequisites: Geometry, Linear Algebra, Analysis
Books:

1. J.W.S Cassels: An introduction to the geometry of numbers, Springer, Berlin, 1972.
2. P.M. Gruber, C.G. Lekkerkerker: Geometry of numbers, North-Holland, 1987.
3. L. Lovász: An algorithmic theory of numbers, graphs, and convexity, CBMS-NSF regional conference series, 1986.

Commitment: 3 hours/week, 3 credits

Contents:
Lattices, sublattices, bases, determinant of a lattice. Convex bodies, elements of the Brunn-Minkowski theory, duality, star bodies. Selection theorems of Blaschke and Mahler. The fundamental theorem of Minkowski, and its generalizations: theorems of Blichfeldt, van der Corput. Successive minima, Minkowski's second theorem. The Minkowski-Hlawka theorem. Reduction theory, Korkine-Zolotarev basis, LLL basis reduction. Connections to the theory of packings and coverings Diophantine approximation: simultaneous, homogeneous, and inhomogeneous, theorems of Dirichlet, Kronecker, Hermite, Khintchin Short vector problem, nearest lattice point problem Applications in combinatorial optimization. The flatness theorem. Covering minima Algorithmic questions, convex lattice polytopes.

71) STOCHASTIC GEOMETRY

Course Coordinator: Imre Bárány
Prerequisites: Geometry, linear algebra, analysis, basic notions of probability theory
Books:

1. L.A. Santalo, Integral geometry and geometric probability, Encyclopedia of Mathematics and its Appl., Vol 1. Addison-Wiley, 1976.
2. J. Pach and P.K. Agarwal, Combinatorial geometry, Academic Press, 1995.
3. C.A. Rogers, Packing and covering Cambridge University Press, 1964.

Commitment: 3 hours/week, 3 credits

Contents:

- Space of lines, measures on the space of lines
- Spaces, groups, measures, intersection formulae
- Minkowski addition and projections
- Lines and flats through convex bodies, the Crofton formulae
- Random polytopes, approximation by random polytopes, expectation of the deviation in various measures
- Connections to floating bodies and affine surface area, extremal properties of balls and polytopes
- Random methods in geometry: the Erdos-Rogers theorem, the Johnson-Lindenstrauss theorem, Dvoretzki's theorem, etc
- Applications in computational geometry

72) BRUNN-MINKOWSKI THEORY

Course Coordinator: Endre Makai
Prerequisites: Geometry, linear algebra, general topology, analysis
Books:

1. T. Bonnesen, W. Fenchel, Theory of convex bodies, BSC Associates, Moscow, Idaho, 1987.
2. R. Schneider, Convex bodies: the Brunn-Minkowski theory, Cambridge Univ. Press, Cambridge, 1993.

Commitment: 3 hours/week, 3 credits
Contents:

- Isoperimetric inequality in the plane, sharpening with the inradius.
- Distance function.
- Support properties, support function.
- Minkowski sum, Blaschke-Hausdorff distance.
- Blaschke selection theorem.
- Almost everywhere differentiability of convex functions.
- Polyhedral approximation.
- Cauchy surface formula.
- Steiner symmetrization, isoperimetric inequality via it.
- Mixed volumes.
- Brunn-Minkowski inequality.

Minkowski's inequality for mixed volumes, isoperimetric inequality

73) HYPERBOLIC MANIFOLDS

Course Coordinator: Gábor Moussong
Prerequisites: Basic Euclidean and projective geometry, elements of group theory and algebraic topology (fundamental groups and covering spaces), basic concepts of differential geometry.
Books:

1. R. Benedetti, C.~Petronio, Lectures on Hyperbolic Geometry, Springer, 1992
2. J. G. Ratcliffe, Foundations of Hyperbolic Manifolds, Springer, 1994

Commitment: 3 hours/week, 3 credits
Contents:

- *Hyperbolic space.* Overview of the projective, quadratic form, and conformal models. Isometries and groups of isometries.
- *Hyperbolic manifolds.* Hyperbolic structures, developing and holonomy, completeness. Discrete groups of isometries of hyperbolic space. The case of dimension two.
- *Constructing hyperbolic manifolds.* Fundamental polyhedra and the Poincaré theorems. Some arithmetic constructions.
- *Mostow Rigidity.* Extending quasi-isometries. The Gromov-Thurston proof of the rigidity theorem for closed hyperbolic manifolds.
- *Structure of hyperbolic manifolds.* Margulis' Lemma and the thick-thin decomposition of complete hyperbolic manifolds of finite volume.
- *Thurston's hyperbolic surgery theorem.* The space of hyperbolic manifolds. Properties of the volume function. Dehn surgery on three-manifolds and Thurston's theorem.
- *The geometrization conjecture.* Topology of three-manifolds: geometric structures and the role of hyperbolic geometry in Thurston's theory.

74) CHARACTERISTIC CLASSES

Course Coordinator: Richárd Rimányi
Prerequisites: Homology theory
Books:

1. J. Milnor, J. Stasheff: Characteristic Classes; Ann. Math. Studies 76, Princeton UP, 1974
2. D. Husemoller: Fibre Bundles; McGraw-Hill, 1966
3. R. M. Switzer: Algebraic Topology-Homotopy and Homology; Springer, 1975

Commitment: 3 hours/week, 3 credits
Contents:

- Differentiable manifolds, maps. Vector bundles. Algebraic manipulations on vector bundles. Examples. Pullback, universal bundle.
- Stiefel-Whitney classes- axiomatic approach. Computations, examples. Application to differentiable topology: embeddings, immersions. Review on Thom polynomial theory and geometric representations of Stiefel-Whitney classes.
- Orientability, Euler class, Almost complex structures, Chern classes. Pontryagin classes.
- Existence questions (review): Cohomology operations; Schubert cells, Schubert calculus; differential geometrical approach.
- Thom isomorphism, Poincaré duality. Applications.
- Characteristic numbers. Unoriented and oriented cobordism groups, computations. Signature formulas.

75) SINGULARITIES OF DIFFERENTABLE MAPS: LOCAL AND GLOBAL THEORY

Course Coordinator: Richárd Rimányi
Prerequisites: Multivariable calculus, algebra.
Books:

1. V. Arnold, S. Gusein-Zade, A. Varchenko: Singularities of Differentiable Maps, Vol. I.; Birkhauser, 1985
2. J. Martinet: Singularities of Differentiable Functions and Maps; London Math. Soc. Lecture Notes Series 58, 1982
3. C.T.C. Wall: Proceedings of Liverpool Singularities, Sypmosium I.; SLNM 192, 1970

Commitment: 3 hours/week, 3 credits
Contents:

- Examples and interesting phenomena about the singularities of differentiable maps. Connection with physics, catastrophes.
- The notion of map germs and jets; the ring of germs of differentiable functions. Modules over this ring. Weierstrass-Malgrange-Mather preparation theorem. Applications.
- Σ^i singularities. Thom-Boardman singularities (examples: fold, cusp, swallow-tail, umbilics), Thom's transversality theorems.
- Useful group actions in singularity theory: A, K. Stability and infinitesimal stability. Finitely determined germs. Classification of stable germs by local algebra. The nice dimensions.

76) FOUR MANIFOLDS AND KIRBY CALCULUS

Course Coordinator: András Stipsicz
Prerequisites: Basic topology, bundles, homology theory
Books:
1. R. E. Gompf, A. Stipsicz: 4-manifolds and Kirby calculus, AMS, 1999.
2. J. Milnor: Lectures on h-cobordism theorem, Princeton Univ. Press, 1965.

Commitment: 3 hours/week, 3 credits

Contents:
- Topological and smooth manifolds, orientations; intersection forms. Classification of bilinear, symmetric, unimodular forms (outline of proof).
- Examples of 4-manifolds, basic computations.
- Handles, handlebodies, handlebody decompositions; Morse functions (existence and generated decomposition).
- Handle slides, handle cancellations. Heegaard and Kirby diagrams (1- and 2-handles). Basic moves of Kirby calculus; the main theorem.
- Further examples: elliptic and ruled surfaces, the blow-up process.
- Realization of intersection forms: Freedman's, Rohlin's, Furuta's and Donaldson's theorems.
- The h-cobordism theorem - outline of proof. Sketch of Freedman's theorem.
- Additional structures on 4-manifolds: spin and spin^c structures, Dirac operators.
- Seiberg-Witten equations, the Seiberg-Witten moduli space and its analytic properties. Proof of Donaldson's theorem.

77) SYMPLECTIC MANIFOLDS, LEFSCHETZ FIBRATION

Course Coordinator: András Stipsicz
Prerequisites: Homology theory, DeRham theory, basic differential topology.
Books:
1. R. E. Gompf, A. Stipsicz: 4-manifolds and Kirby calculus, AMS, 1999.
2. J. Morgan: The Seiberg-Witten equations and applications to the topology smoth 4-manifolds, Princeton, 1996.

Commitment: 3 hours/week, 3 credits

Contents:
- Symplectic, almost complex and Kahler structures, Noether formula.
- Constructions of symplectic manifolds. Obstructions to the existence of a symplectic form.
- Lefschetz fibrations, surface bundles. Monodromies, mapping class groups.
- Thurston's and Gompf's theorem.
- Lefschetz pencils, Donaldson's theorem.
- Spin^c structures, Dirac operators.
- Seiberg-Witten equations; the equations on Kahler surfaces.
- Seiberg-Witten invariants, basic classes; Taubes' perturbation.
- Pseudo-holomorphic submanifolds, Taubes' theorem and some consequences of it.

78) ADVANCED INTERSECTION THEORY

Course Coordinator: Endre Szabó

Prerequisites: Basic Algebraic Geometry and Language of Schemes courses. Familiarity with Local Algebra is an advantage.

Books:
W. Fulton: Intersection Theory. Springer, 1986.

Commitment: 3 hours/week, 3 credits

Contents:

- Cycles on algebraic varieties, Chow groups. Functorial properties of Chow groups.
- Construction of an intersection product on cycles via moving lemmas and/or Fulton's deformation technique.
- Vector bundles and their characteristic classes. Splitting principle.
- The Grothendieck K-group (resp. G-group) of locally free (resp. coherent) sheaves. Comparison of the two for regular projective schemes (by applying methods of local algebra: finite locally free resolutions, Koszul complexes). Topological filtration on K(X), relation to Chow groups.
- The Grothendieck-Riemann-Roch theorem for smooth varieties, with applications.
- A survey of advanced topics (optional): relation to higher K-theory, motivic cohomology, Arakelov geometry.

79) DESCRIPTIVE SET THEORY

Course Coordinator: Péter Komjáth

Prerequisites: -

Books:
1. K. Kuratowski: Topology, Academic Press, 1968.
2. A.S. Kechris: Classical Descriptive Set Theory, Springer, 1995.

Commitment: 3 hours/week, 3 credits

Contents:

Borel, analytic, projective sets, universality, reduction, separation theorems, ranks, scales, games, axiom of determinancy, large cardinals, trees.
Forcing.

80) ADVANCED SET THEORY

Course Coordinator: Soukup Lajos
Prerequisites: -
Books:

1. T. Bartoszynski and H. Judah, Set theory on the structure of the real line, A K Peters, 1995.
2. Thomas Jech, Set theory, Spinger-Verlag, 1997.
3. István Juhász, Cardinal functions in topology - ten years later. Amsterdam: Mathematisch Centrum, 1980.
4. Akihiro Kanamori, Higher Infinite, Springer-Verlag, 1994.
5. Kenneth Kunen: Set theory. An introduction to Independence Proofs, Elsevier, 1999.

Commitment: 3 hours/week, 3 credits
Contents:
Iterated forcing and preservation theorems. Finite support iterations and Martin's Axiom. Countable support iterations and PFA. Permutation models. Combinatorial set-theory and applications to topology. Large cardinals. Basic pcf theory with applications to algebra and to topology. Set theory of the reals. ZFC results. Cardinal invariants. Forcing constructions. Independence results in topology
Determinacy. Infinite games and combinatorics.

81) LOGICAL SYSTEMS

Course Coordinator: Andreka Hajnal
Prerequisites: Introduction to Mathematical logic.
Books:

1. J. Barwise and S. Feferman, editors, Model-Theoretic Logics, Springer-Verlag, Berlin, 1985.
2. W.J. Blok and D.L. Pigozzi: Algebraizable Logics, Memoirs AMS, 77, 1989.
3. L. Henkin, J.D. Monk, and A. Tarski: Cylindric Algebras, North-Holland, Amsterdam, 1985.

Commitment: 3 hours/week, 3 credits

Contents:
Establishing a meta-theory for investigating logical systems (logics for short), the concept of a general logic, some distinguished properties of logics. Filter-property (syntactical) substitution property. Semantical substitution property. Structurality. Algebraizability. Algebraization of logics. Linden-baum-Tarski algebras. Characterization theorems for completeness, soundness and their algebraic counterparts; concepts of compactness and their algebraic counterparts; definability properties and their algebraic counterparts; properties and their algebraic counter-parts; omitting types properties and their algebraic counterparts.
Applications, examples; propositional logic; (multi-)modal logical systems; dynamic logics (logics of actions, logics of programs etc.)
Connections with abstract model theory; elements of Abstract Model Theory (AMT); absolute logics; Abstract Algebraic Logic (AAL); Lindstrom's theorem in AMT versus that in AAL

82) SET-THEORETIC TOPOLOGY

Course Coordinator: István Juhász
Prerequisites: Modern Set-Theory, Advanced Set-Theory
Books:

1. K. Kunen, J.E. Vaughan, Handbook of Set-Theoretic Topology, Noth-Holland, 1995.
2. Miroslav Huvsek and Jan van Mill. Recent progress in general topology. North-Holland , 1992.

Commitment: 3 hours/week, 3 credits

Contents:

- Cardinal functions and their interrelationships
- Cardinal functions on special classes, in particular on compact spaces
- Independence results, consequences of CH, Diamond, MA and PFA
- Topological results in special forcing extensions, in particular in Cohen models
- S and L spaces, HFD and HFC type spaces

83) LOGIC AND RELATIVITY

Course Coordinator: Németi István
Prerequisites: Familiarity with the basics of first order logic, e.g. formulas, models, satisfaction, validity. The notion of a first order theory and its models.
Books:

1. Andréka, H., Madarász, J., Németi, I.., Andai, A. Sain, I., Sági, G., Tőke, Cs.: Logical analysis of relativity theory. Parts I-IV. Lecture Notes. www.math-inst.hu/pub/algebraic-logic.
2. d'Inverno, R.: Introducing Einstein's Relativity. Clarendon Press, Oxford, 1992.
3. Goldblatt, R.: Orthogonality and spacetime geometry. Springer-Verlag, 1987.

Commitment: 3 hours/week, 3 credits

Contents:

- Axiomatizing special relativity purely in first order logic. (Arguments from abstract model theory against using higher order logic for such an axiomatization.)
- Proving some of the main results, i.e. "paradigmatic effects", of special relativity from the above axioms. (E.g. twin paradox, time dilation, no FTL observer etc.)
- Which axiom is responsible for which "paradigmatic effect"
- Proving the paradigmatic effects in weaker/more general axiom systems (for relativity).
- Applications of definability theory of logic to the question of definability of "theoretical concepts" from "observational ones" in relativity. Duality with relativistic geometries.
- Extending the theory to accelerated observers. Acceleration and gravity. Black holes, rotating (Kerr) black holes. Schwarzschild coordinates, Eddington-Finkelstein coordinates, Penrose diagrams. Causal loops (closed time-like curves). Connections with the Church-Turing thesis.

84) FRONTIERS OF ALGEBRAIC LOGIC

Course Coordinator: Andréka Hajnal
Prerequisites: Algebraic logic and model theory course.
Books:
1. Henkin, L. Monk, J. D. Tarski, A. Andréka, H. Németi, I.: Cylindric Set Algebras. Lecture Notes in Mathematics Vol 883, Springer-Verlag, Berlin, 1981.
2. Henkin, L. Monk, J. D. Tarski, A.: Cylindric Algebras Part II. North-Holland, Amsterdam, 1985.
3. Andréka, H., Németi, I., Sain, I.: Algebraic Logic. Chapter in Handbook of Philosophical Logic, second edition. Kluwer.
4. van Benthem, J.: Exploring Logical Dynamics. Studies in Logic, Language and Information, CSLI Publications, 1996.

Commitment: 3 hours/week, 3 credits

Contents:
- Re-thinking the role of algebraic logic in logic. Theories as algebras, interpretations between theories as homomorphisms.
- The finitization problem, its connections with finite model theory.
- Parallels and differences between algebraic logic and (new trends in) the modal logic tradition.
- Connections and differences between the algebraic logic based approach and abstract model theory (e.g. in connection with the Lindström type theorems).
- Tarskian representation theorems and duality theories in algebraic logic and their generalizations (e.g. in axiomatic geometry and relativity theory).

85) CLASSICAL ANALYTIC NUMBER THEORY

Course Coordinator: János Pintz
Prerequisites: Complex function theory
Books:
1. G. Tenenbaum, Introduction to analytic and probabilistic number theory, Cambridge studies in advanced mathematics 46, Cambridge University Press 1995.
2. H. Davenport, Multiplicative number theory (2nd ed.) Springer, Graduate texts in mathematics 74, 1980.

Commitment: 3 hours/week, 3 credits

Contents:
- Dirichlet series. Riemann's zeta function, its analytic continuation, functional equation and product formulas. Inversion for the Mellin transform.
- Exact prime number formulas with integral, and with the roots of the zeta function. The prime number theorem with the classical remainder term.
- L-functions. Siegel's theorem. The prime number theorem for arithmetical progressions.

86) PROBABILISTIC NUMBER THEORY

Course Coordinator: Imre Ruzsa
Prerequisites: Probability theory, Complex function theory.
Books:

1. G. Tanenbaum, Introduction to analytic and probabilistic number theory, Cambridge studies in advanced mathematics 46, Cambridge University Press 1995, part III
2. P. D. T. A. Elliott, Probabilistic number theory I-II, Springer, Grundlehren der Mathematischen Wissenschaften 239-240, 1979, 1980.

Commitment: 3 hours/week, 3 credits

Contents:

- Statistical properties of arithmetical functions: average, extremal orders, limiting distribution.
- The Erdős-Wintner theorem. The Erdős-Kac theorem and generalizations. Kubilius's model. The analytic method of Halász.

87) PROBABILISTIC NUMBER THEORY, LEVEL 2

Course Coordinator: Imre Ruzsa
Prerequisites: Probability theory, Complex function theory
Books: There are no textbooks for these subjects, the original papers have to be used.

Commitment: 3 hours/week, 3 credits

Contents:
This continues the study of additive and multiplicative functions, primarily with analytic methods, Halász's method and refinements. Topics: Concentration estimates for additive functions. Properties of the distribution function of additive functions: continuity, absolute continuity. Distribution of additive functions on shifted primes

88) MODERN PRIME NUMBER THEORY

Course Coordinator: János Pintz
Prerequisites: Complex function theory.
Books: There are no textbooks for these subjects, the original papers have to be used.

Commitment: 3 hours/week, 3 credits

Contents:

- Zero-free regions for zeta and L-functions. Density estimates for the roots in the critical strip. Estimates for the difference of consecutive primes and for the number of primes in short intervals.
- Turán's power sum method.
- Omega-estimates for the remainder terms for primes and for related sums with the Möbius and Liouville functions.

89) EXPONENTIAL SUMS IN COMBINATORIAL NUMBER THEORY

Course Coordinator: András Sárközy
Prerequisites: Harmonic Analysis
Books: There are no textbooks for these subjects, the original papers have to be used.
Commitment: 3 hours/week, 3 credits
Contents:
We learn to use Fourier-analytic techniques to solve several problems on general sets of integers. In particular: to find estimates for sets free of arithmetic progressions; methods of Roth, Szemerédi, Bourgain and Gowers. To find arithmetic progressions and Bohr sets in sumsets: methods of Bogolyubov, Bourgain, and Ruzsa's construction. Difference sets and the van der Corput property.

90) INFORMATION THEORETIC METHODS IN MATHEMATICS

Course Coordinator: Katalin Márton
Prerequisites: Probability and Statistics ; Measure and integral; measure preserving maps.
Books:
1. T.M. Cover & J.A. Thomas: Elements of Information Theory. Wiley, 1991.
2. I. Csiszar: Information Theoretic Methods in Probability and Statistics. IEEE Inform. Th. Soc. Newsletter, 48, March 1998.
3. G. Simonyi: Graph entropy: a survey. In: Combinatorial Optimization, DIMACS Series on Discrete Mathematics and Computer Science, Vol. 20, pp. 399-441, 1995.

Commitment: 3 hours/week, 3 credits

Contents:
Applications of information theory in various fields of mathematics are discussed.
Probability:
- Dichotomy theorem for Gaussian measures.
- Sanov's large deviation theorem for empirical distributions, and Gibbs' conditioning principle.
- Measure concentration.

Statistics:
- Wald inequalities.
- Error exponents for hypothesis testing.
- Iterative scaling, generalized iterative scaling, and EM algorithms.
- Minimum description length inference principle

Combinatorics:
- Using entropy to prove combinatorial results.
- Graph entropy and its applications.
- Graph capacities (Shannon, Sperner), applications.
- Ergodic theory:
- Kolmogorov--Sinai theorem.

Information theoretic proofs of inequalities.

91) SELECTED TOPICS IN PROBABILITY

Course Coordinator: Péter Major

Prerequisites: Stochastics, Introduction to Probability and Statistics; Measure and integration; Fourier integral.

Books:

1. L. Breiman: Probability. Addison-Wesley, Reading, Massachusetts , 1968
2. M. Csörgő-P. Révész: Strong Approximations in Probability and Statistics. Academic Press, New York , 1981.
3. P. Major: Series of Problems in Probability Theory, http://www.renyi.hu/\~{}major.

Commitment: 3 hours/week, 3 credits

Contents:

- The central limit theorem and Fourier analysis.
- General limit theorems for sums of independent random variables and infinitely divisible distributions.
- Large deviations.
- Wiener process.
- Markov processes.
- Poisson process.
- Invariance principles in Probability, strong approximations.

92) INVARIANCE PRINCIPLES IN PROBABILITY AND STATISTICS

Course Coordinator: Pál Révész

Prerequisites: Stochastics, Probability; elements of functional analysis.

Books:

1. M. Csörgő-P. Révész: Strong Approximations in Probability and Statistics. Academic Press, New York ,1981.
2. P. Révész: Random Walk in Random and Non-Random Environments. World Scientific, Singapore , 1990.
3. M. Csörgő-L. Horváth: Weighted Approximations in Probability and Statistics. Wiley, New York , 1993.

Commitment: 3 hours/week, 3 credits

Contents:

- Functional central limit theorem. Donsker's theorem via Skorokhod embedding. Weak convergence in D[0,1].
- Strassen's strong invariance theorem.
- Strong approximations of partial sums by Wiener process: Komlós-Major-Tusnády theorem and its extension (Einmahl, Sakhanenko, Zaitsev).
- Strong invariance principles for local time and additive functionals. Iterated processes.
- Strong approximation of empirical process by Brownian bridge: Komlós- Major-Tusnády theorem.
- Strong approximation of renewal process.
- Strong approximation of quantile process.
- Asymptotic results (distributions, almost sure properties) of functionals of the above processes.

93) STOCHASTIC PROCESSES

Course Coordinator: István Berkes
Prerequisites: Information Theory, Probability, Functional Analysis.
Books:
1. A.V. Skorokhod: Studies in the Theory of Random Processes. Addison-Wesley, Reading, Massachusetts , 1965.
2. W. Feller: An Introduction to Probability Theory and its Applications, Vol. II. Second edition. Wiley, New York , 1971.
3. D. Revuz-M. Yor: Continuous Martingales and Brownian Motion. Third edition. Springer, Berlin , 1999.

Commitment: 3 hours/week, 3 credits

Contents:
- Random walk., Renewal processes.
- Markov chains. Transition probabilities. Recurrence, ergodicity. Existence of stationary distribution.
- Processes with independent increments. Lévy processes. Stable processes. Stationary processes. Ergodicity. Bochner-Khintchine theorem. Markov processes. Infinitesimal generator. Chapman-Kolmogorov equations.
- Branching processes. Asymptotic results. Birth and death process.
- Martingales. Stopping times. Maximal inequalities. Martingale convergence theorems. Quadratic variation. Burkholder-Davis-Gundy inequalities.

94) STOCHASTIC ANALYSIS

Course Coordinator: József Fritz
Prerequisites: Information Theory, Probability Theory, elements of functional analysis.
Books:
1. N. Ikeda and S. Watanabe: Stochastic Differential Equations and Diffusion Processes. North-Holland, Amsterdam , 1981, 1989.
2. D.W. Stroock and S.R.S. Varadhan: Multidimensional Diffusion Processes. Springer, Berlin , 1979.
3. T.M. Liggett: Interacting Particle Systems. Springer, Berlin , 1983.

Commitment: 3 hours/week, 3 credits

Contents:
- Generalities on stochastic processes with continuous parameter. Kolmogorov's existence theorem, and his criterion for the continuity of sample paths.
- Poisson point process and fields, construction of jump Markov processes. Wiener process, construction and basic properties. Itô integral and Itô formula. Solution of the simplest stochastic differential equations.
- Martingales and stopping times, basic inequalities. Martingales with continuous

time, the Doob-Meyer decomposition. Quadratic variation and stochastic integrals. The general Itô formula.
- Martingale characterization of Wiener and Poisson processes. Representation theorems for martingales and for processes with independent increments. The Girsanov theorem.
- Stochastic differential equations, strong and weak solutions, existence and uniqueness of strong solutions. The martingale approach, applications of Girsanov's theorem to weak solutions.
- Generators of Markov processes, the Hille-Yoshida theory of Markov semigroups. Perturbation theorems and convergence of semigroups. Uniqueness of the martingale problem. Diffusion processes.
- Construction of interacting Markov processes. Stochastic Ising models and exclusion process, large scale behaviour. Entropy methods, logarithmic Soboliev inequalities. Applications to image recognition.
- Foundations of Mathematical Finance. The fundamental theorems of asset pricing, Black-Scholes and related models. Admissible and optimal strategies. Incomplete models, examples.

95) PATH PROPERTIES OF STOCHASTIC PROCESSES

Course Coordinator: Pál Révész

Prerequisites: Invariance Principles in Probability and Statistics, Stochastic Processes.

Books:

1. M. Csörgő-P. Révész: Strong Approximations in Probability and Statistics. Academic Press, New York , 1981.
2. P. Révész: Random Walk in Random and Non-Random Environments. World Scientific, Singapore , 1990.
3. P. Révész: Random Walks of Infinitely Many Particles. World Scientific, Singapore , 1994.
4. D. Revuz-M. Yor: Continuous Martingales and Brownian Motion. Third edition. Springer, Berlin , 1999.

Commitment: 3 hours/week, 3 credits

Contents:

- Constructions of Wiener process. Modulus of continuity.
- Laws of the iterated logarithm. Strassen's theorem.
- Increments of Wiener process.
- Local times, additive functionals and their increments.
- Asymptotic properties, invariance principles for local time and additive functionals. Dobrushin's theorem.
- Path properties of random walks, their local times and additive functionals.
- Random walk in random environment.
- Random walk in random scenery.
- Branching random walk and branching Wiener process.
- Almost sure central limit theorems.

96) NONPARAMETRIC STATISTICS

Course Coordinator: Endre Csáki
Prerequisites: Probability, Mathematical Statistics.
Books:
1. L. Takács: Combinatorial Methods in the Theory of Stochastic Processes. Wiley, New York , 1967.
2. J. Hájek: Nonparametric Statistics. Holden-Day, San Francisco , 1969.

Commitment: 3 hours/week, 3 credits
Contents:
- Order statistics and their distribution.
- Empirical distribution function. Glivenko-Cantelli theorem and its extensions.
- Estimation of the density function. Kernel-type estimators.
- U-statistics.
- Rank correlation. Kendall-s tau.
- Nonparametric tests: goodness of fit, homogeneity, independence.
- Empirical process, approximation by Brownian bridge.
- Komlós-Major-Tusnády theorem.
- Tests based on empirical distribution: Kolmogorov-Smirnov, von Mises tests.
- Quantile process. Bahadur-Kiefer process.
- Rank tests. Wilcoxon-Mann-Whitney test.

97) MULTIVARIATE STATISTICS

Course Coordinator: Gábor Tusnády
Prerequisites: Probability; Mathematical statistics; Linear Algebra.
Books:
1. T.W. Anderson: An Introduction to Multivariate Statistical Analysis. Wiley, New York (1958).
2. K.V. Mardia-J.T. Kent-J.M. Bibby: Multivariate Analysis. Academic Press, New York (1979).

Commitment: 3 hours/week, 3 credits
Contents:
- Multivariate normal distribution. Moments, correlation, partial correlations. Conditional and marginal distributions. Regression. Empirical moments, maximum likelihood estimation of the parameters. Wishart matrix and its distribution.
- Testing for independence.
- Least squares. Various regression methods (linear, polynomial, orthogonal, spline).
- Variance and covariance analysis.
- Linear models.
- Design of experiments.
- Analysis of contingency tables.
- Time series. ARMA processes.
- Factor analysis.

98) INFORMATION THEORETICAL METHODS IN STATISTICS

Course Coordinator: Imre Csiszár
Prerequisites: Probability; Mathematical Statistics; Information Theory.
Books:

1. I. Csiczár: Lecture Notes, University of Meryland , 1989.
2. J. Kullback: Information Theory and Statistics. Wiley, 1989.
3. J. Rissanen: Stochastic Complexity in Statistical Inquiry. World Scientific, 1989.

Commitment: 3 hours/week, 3 credits
Contents:

- Definition and basic properties of Kullback I-divergence, for discrete and for general probability distributions.
- Hypothesis testing: Stein lemma, Wald inequalities. Sanov's large deviation theorem, error exponents for testing simple and composite hypotheses (discrete case).
- Estimation: performance bounds for estimators via information-theoretic tools.
- Hájek's proof of the equivalence or orthogonality of Gaussian measures. Sanov's theorem, general case.
- Information geometry, I-projections. Exponential families, log-linear models. Iterative scaling, EM-algorithm. Gibbs conditioning principle.
- Information theoretic inference principles: Maximum entropy, maximum entropy on the mean, minimum description length. The BIC model selection criterion; consistency of BIC order estimation.
- Generalizations of I-divergence: f-divergences, Bregman distances.

99) NUMERICAL METHODS IN STATISTICS

Course Coordinator: Gábor Tusnády
Prerequisites: Probability; Mathematical Statistics.
Books:

1. W. Freiberger-U. Grenander: A Course in Computational Probability and Statistics. Springer, New York (1971).
2. J.E. Gentle: Random Number Generation and Monte Carlo Methods. Springer, New York (1998).

Commitment: 3 hours/week, 3 credits
Contents:

- Combinatorial algorithms with statistical applications.
- Numerical evaluation of distributions.
- Generating random numbers of various distribution.
- Computer evaluation of statistical procedures. Estimation methods. Robust procedures. Testing statistical hypothesis. Testing for normality.
- Sequential methods.
- EM algorithm.
- Statistical procedures for stochastic processes. Time series. Reliability. Life tests.
- Bootstrap methods.
- Monte Carlo methods
- Statistical software packages

100) ERGODIC THEORY AND DYNAMICAL SYSTEMS

Course Coordinator: Domokos Szász
Prerequisites: Probability, Measure and Integration.
Books:
1. I.P. Cornfeld, S. V. Fomin and Ya. G. Sinai: Ergodic Theory. Springer, Berlin (1982).
2. A. Katok and B. Hasselblatt, Introduction to the Modern Theory of Dynamical Systems. Cambridge University Press (1995).
Commitment: 3 hours/week, 3 credits
Contents:

- Fixed point theorems. Applications
- Variational principles and weak convergence. The n-th variation. Necessary and sufficient conditions for local extrema. Weak convergence. The generalized Weierstrass existence theorem. Applications to calculus of variations. Applications to nonlinear eigenvalue problems. Applications to convex minimum problems and variational inequalities. Applications to obstacle problems in Elasticity. Saddle points. Applications to duality theory. The von Neumann Minimax theorem on the existence of sadle points. Applications to game theory.
- Nonlinear monotone operators. Applications.

101) ERGODIC THEORY AND COMBINATORICS

Course Coordinator: Gábor Elek
Prerequisites: Real Analysis; Measure Theory.
Books:
H. Furstenberg: Recurrence in Ergodic Theory and Combinatorial Number Theory. Princeton University Press, Princeton, N.J. (1981).

Commitment: 3 hours/week, 3 credits

Contents:

- Compact systems, recurrence, transitivity, minimality.
- The Birkhoff recurrence theorem.
- Van der Waerden's theorem.
- The Birkhoff Pointwise Ergodic Theorem and the Mean Ergodic Theorem.
- The existence of invariant measures.
- Ergodic measures and extremal points.
- Sárközy's theorem and Weyl's equidistribution formula.
- Weakly mixing transformations.
- The proof of Roth's theorem.
- Conditional expectation, disintegration of measures.
- The Furstenberg-Katznelson structure theorem.
- The proof of Szemerédi's theorem.
- Ultrafilters.
- Hindman's theorem.

102) DATA COMPRESSION

Course Coordinator: Imre Csiszár

Prerequisites: Information theory

Books:

1. T.M. Cover-J.A. Thomas: Elements of Information Theory. Wiley, New York (1991).
2. I. Csiszár: Lecture Notes, University of Maryland (1989).
3. K. Sayood: Introduction to Data Compression. Morgan-Kauffmann, San Francisco (1996).
4. D. Salomon: Data Compression: the Complete Reference. Springer, New York (1998).

Commitment: 3 hours/week, 3 credits

Contents:

Lossless compression:

- Shannon-Fano, Huffman and arithmetic codes. "Ideal codelength", almost sure sense asymptotic optimality. Data compression standard for fax.
- Optimum compression achievable by block codes. Information sprectum, information stability, Shannon--McMillan theorem.
- Universal coding. Optimality criteria of mean and max redundancy; mixture and maximum likelihood coding distributions. Mathematical equivalence of minimax redundancy and channel capacity. Krichevsky-Trofimov distributions, minimax redundancy for memoryless and finite state sources.
- The context weighting method. Burrows-Wheeler transform. Dictionary-type codes. Lempel-Ziv codes, their weak-sense universality for stationary ergodic sources.
- Universal coding and complexity theory, Kolmogorov complexity, finite state complexity, Ziv complexity.

Lossy compression:

- Shannon's rate distortion theorem for discrete memoryless sources, and its universal coding version. Extensions to continuous alphabets and sources with memory. Rate-distortion function of Gaussian sources.
- Uniform and non-uniform scalar quantization, Max-Lloyd algorithm. Companded quantization, Bennett's integral.
- Vector quantization: Linde-Buzo-Gray algorithm, tree structured vector quantizers, lattice vector quantizers. Transform techniques. Pyramidal coding. The JPEG image compression standard.

103) CRYPTOLOGY

Course Coordinator: Tibor Nemetz

Prerequisites: Mathematical Statistics, Information Theory.

Books:

1. S.W. Golomb: Shift Register Sequences. Holden Day, San Francisco (1960).
2. M.E. Hellman: An Extension of the Shannon Theory Approach to Cryptography, IEEE-IT 23 (1977), 289-294.
3. D. Kahn: The Codebreakers. MacMillan, New York (1967).
4. C.E. Shannon: Communication Theory of Secrecy Systems, Bell Syst. Techn. J. 28 (1949), 656-715.
5. G.J. Simmons: Contemporary Cryptology: The Science of Information Security. IEEE Press, New York (1992).

Commitment: 3 hours/week, 3 credits

Contents:

- First steps: From Caesar to the simple substitution. Transposition and its statistical solution. Language statistics, use of statistical hypothesis testing. Intuitive notion of the unicity distance. Definition of codes.
- Poly-alphabetic codes: Vigen\'ere-algorithm. Statistical determination of the length of the key-word. Statistical solutions. The role of cryptographic keys.
- The one time pad. Recognizing double use of key-stream: test of coincidence. Solving the two-times pad: The Kerkhoff method. Learning from the Venona papers. Solving by side-information.
- Data compression and its utilization in cryptography. Dictionary look-up compression, and preparing dictionaries. Ref: parallel course on Data Compression. Generating random and pseudo-random sequences.
- Stream chipers. Short introduction to LFSRs, Ref: Golomb (1960). Linear cryptanalysis. Encryption machines.
- Shannon theory and its extension by Hellman. Notion of redundancy. Ref: Shannon (1949), Hellman (1977).
- DES. Triple DES. Differential cryptanalysis. AES.
- Introduction to public key cryptography and to security questions of public networks. Ref: Simmons (1992). The RSA algorithm.
- Applications of public key cryptography. Trusted third party, CA.
- Digital signature. Hash functions and algorithms. The SHA-1 standard. Critics of CRC-hashing.
- Cryptographic protocols. Zero-knowledge protocols. Secret sharing.
- Digital fingerprint, digital watermarks.

104) COMBINATORIAL OPTIMIZATION

Course Coordinator: András Frank
Prerequisites: Elements of graph theory and linear algebra
Books:
B. Korte and J. Vygen : Combinatorial Optimization , Springer, 2000.
Commitment: 3 hours/week, 3 credits
Contents:

- Linear programming: polyhedron, polytope, cone. Farkas Lemma, Duality theorem. Fourier Motzkin elimination, Simplex and ellipsoid methods.
- Totally dual matrices and their application in optimization: Konig-Egervary theorem, the Hungarian method. Min-cost paths. Flow problems: Max-flow min-cut, minimum cost flows. Gomory-Hu trees.
- Characterization of integral polyhedra: the Hoffman-Edmonds-Giles theorem. Totally dual integral systems. submodular flows and their applications: Lucchesi Younger theorem, Nash-Williams orientation theorem. The matching polyhedron, algorithm for finding maximum weight matchings.
- Matroid partition and intersection theorem, submodular functions. Applications to connectivity problems. Weighted matroid intersection algorithm.
- Finding minimum cost arborescences (Fulkerson's algorithm) and disjont arborescences (Edmonds' theorem).
- Disjoint paths problems, theorem of Okamura and Seymour. Minimum cardinality T-jons, packing T-cuts (Seymour's theorem)

105) QUANTUM COMPUTING

Course Coordinator: Vince Grolmusz
Prerequisites: An "Algorithms and complexity" type course, including the solid understanding of randomized and deterministic Turing machines.
Books:
J. Gruska, Quantum Computing, McGrawHill, 1999.
Commitment: 3 hours/week, 3 credits
Contents:

- The comparison of probabilistic and quantum Turing Machines. Probabilities vs. complex amplitudes. Positive interference and negative interference. Why complex amplitudes? Background in Physics: some experiments with bullets and electrons.
- Background in Mathematics. Linear algebra, Hilbert spaces, projections. Observables, measuring quantum states.
- The qubit. Tensor product of vectors and matrices. Properties of tensor product. Two qubit registers. Quantum entanglement, examples. n-qubit registers.
- The Fourier transform. Quantum parallelism. Van Dam's algorithm, Deutsch's problem. The deterministic solution of Deutsch's problem.
- The promise problem of Deutsch and Józsa.
- Simon's Problem.
- Grover's database-search algorithm. Lower bound for the database-search problem.
- Shor's integer factoring algorithm.
- Complexity theoretic results: BQP is in PSPACE, P is in BQP, BPP is in BQP.
- Results in Quantum Communication Complexity, Quantum Cryptography

106) COMPUTATIONAL GEOMETRY

Course Coordinator: Géza Tóth
Prerequisites: Geometry, Basic linear algebra, Algorithms, Discrete mathematics
Books:
M. de Berg, M. van Kreveld, M. Overmars, and O. Schwarzkopf: Computational Geometry - Algorithms and Applications, Springer, Berlin, 1997.
Commitment: 3 hours/week, 3 credits
Contents:

- Line segment intersection
- Convex hull
- Polygon triangulation, art gallery problems
- Linear programming
- Range searching
- Point location
- Voronoi diagrams, Delaunay triangulations
- Arrangements and duality
- Geometric data structures
- Motion planning
- Visibility graphs, ray shooting
- Applications in Computer Science, Robotics, Computer graphics, Geometric optimization

107) RANDOM COMPUTATION

Course Coordinator: Miklós Ruszinkó
Prerequisites: Knowledge of the basic notions in combinatorics, probability theory and complexity theory.

Books:
1. M. Rajeev; P. Raghavan, Randomized algorithms, Cambridge University Press, Cambridge, 1995.
2. M. Dyer, A. Frieze, M. Jerrum, Approximate Counting and Rapidly Mixing Markov Chains, in preparation.

Commitment: 3 hours/week, 3 credits

Contents:

General description*:* The Monte Carlo method is an approach for estimating quantities which
are hard to compute exactly. The quantity of interest z is expressed as the expectation z=E(Z) of the random variable Z over some probability space Ω. It is assumed that some efficient procedure for sampling from Ω is available. The quantity of interest z is estimated by taking the mean of sufficiently many independent samples of Z. The goal of this course is to present and analize important algorithms based on the above ideas.

Topics covered:

- Min-cut algorithm, Las Vegas and Monte Carlo, binary planar partitions, probabilistic recurrences,
  computation models and complexity classes, Randomness and non-uniformity.
- Graphs and eigenvalues
- Peron theory, Spectral gap
- Random walks on abstract graphs
- Markov chains and random walks, 2-SAT: an example, random walks on graphs, electrical networks and conductance, cover times, expanders and rapidly mixing random walks, probability amplification by random walks on expanders.
- Algorithms using random walks: How to approximate the permanent, the number of mathcings, Hamilton paths and Eulerian orientations?
- Computing the volume of a convex body.
- On-line algorithms : Competitive analysis, the paging problem, adversary models, paging against an oblivious adversary, relating the adversaries, the adaptive on-line adversary, the k-server problem

108) LOGIC OF PROGRAMS

Course Coordinator: László Csirmaz
Prerequisites: Basics of Formal Logic, Introduction to Computer Science, Computer Languages

Books:
1. Time and Logic, a computational approach. Bolc, L., Szalas, A., editors, UCL-press ltd., London, 1995.
2. van Benthem, J., A note on dynamic arrow logics. In J. van Eijck, editor, Logic and Information flow. Kluwer, Dordrecht, 1993.
3. Pratt, V.R., Dynamic algebras as a well behaved fragment of relation algebras. In Algebraic logic and universal algebra in computer science, vol. 425 of Lecture Notes in Computer Science, pp. 77-110. Springer-Verlag, Berlin, 1990.
4. Abadi, M., Burrows, Needham, Logic of authentization. publisher: Digital (Systems research center, Palo Alto, California/94301).
5. van Benthem, J., Exploring logical dynamics. Studies in Logic, Language and Information, CSLI publications, 1995.

Commitment: 3 hours/week, 3 credits

Contents:
Dynamic logic, reasoning about actions and programs.
Temporal logics of programs.
Distinguished methods (logics) for proving program properties and their model theoretic characterisations.
Provability and unprovability of properties of programs; ultraproduct method.
Provability and unprovability of properties of programs; ultraproduct method.
Connections with Peano's Arithmetic.
Further applications of many-sorted modal logic in computer science e.g. logics of authentizations (or protocols).
Connections with recent trends in modal logic.

109) TOPICS IN MATHEMATICAL ANALYSIS

Course Coordinator: Gheorghe Morosanu
Prerequisites: Undergraduate Calculus, Elementary Linear Algebra
Books:

1. M.H. Protter and C.B. Morrey, A first course in real analysis, 2nd Edition, Spinger-Verlag, 1977.
2. E. Zeidler, Applied Functional Analysis, Appl. Math. Sci. 108, Springer-Verlag, 1995.

Commitment: 3 hours/week, 3 credits

Contents:

- Various classes of functions
- Integration Theory
- L^p – spaces
- Generalized functions
- Fourier series
- Elements of variational calculus
- Typical exercises and specific applications to Optimization, Physics, etc.

110) CALCULUS OF VARIATIONS AND OPTIMAL CONTROL

Course Coordinator: Gheorghe Morosanu
Prerequisites: Calculus, Linear Algebra
Books:
1. E.B. Lee and L. Markus, Foundations of Optimal Control Theory, John Wiley, 1967.
2. G. Ye. Shilov, Mathematical Analysis, Pergamon Press, 1965.

Commitment: 3 hours/week, 3 credits

Contents:

- Classical examples of variational problems
- Differentiable functionals
- Euler-Lagrange equations
- Examples of optimal control problems
- Optimal control of linear systems
- Optimal control for linear processes with integral convex cost criteria
- Necessary and sufficient conditions for optimal control
- Applications